Reports of the Department of Geodetic Science

Report No. 82

PREPROCESSING OPTICAL SATELLITE OBSERVATIONS

by

Frank Donald Hotter

Prepared for

National Aeronautics and Space Administration
Washington, D.C.

Contract No. NSR 36-008-033
OSURF Project No. 1997

The Ohio State University
Research Foundation
Columbus, Ohio 43212

April, 1967

PREFACE

This project is under the direction of Professor Ivan I. Mueller of the Department of Geodetic Science, The Ohio State University. Project manager is Jerome D. Rosenberg, Geodetic Satellites, Code SAG, NASA Headquarters, Washington, D.C. The contract is administered by the Office of Grants and Research Contracts, Office of Space Science and Applications, NASA Headquarters, Washington, D.C.

The report was written by Frank D. Hotter in partial fulfillment of the requirements for the Master of Science degree at The Ohio State University, and it has been revised and amended by Professor Mueller who served as his faculty adviser.

Reports related to NASA Contract No. NSR 36-008-033 and published to date are the following:

<u>The Determination and Distribution of Precise Time</u>, Reort No. 70 of the Department of Geodetic Science, The Ohio State University.

<u>Proposed Optical Network for the National Geodetic Satellite Program</u>, Report No. 71 of the Department of Geodetic Science, The Ohio State University.

<u>Preprocessing Optical Satellite Observations</u>, Report No. 82 of the Department of Geodetic Science, The Ohio State University.

<u>Quarterly Progress Reports</u>, Numbers 1, 2, 3, 4, 5, 6.

ABSTRACT

The purpose of the report is to examine the basic theory of stellar triangulation; review the procedures that four U. S. agencies currently use in reducing geodetic satellite observations for the National Geodetic Satellite Program; recommend preprocessing procedures to homogenize the data from the agencies for further geodetic studies; and finally recommend that additional data as specified in the report be provided for the National Aeronautics and Space Administration so that the data from various agencies may be correlated.

ACKNOWLEDGEMENTS

A great deal of the material presented in the report was gathered during a trip to the various agencies in December 1966. The trip was funded by the National Aeronautics and Space Administration Contract Number NSR 30-008-003.

The author wishes to express his sincere appreciation to the many persons in the agencies who took time from their busy schedules to aid in the compilation of material necessary for this text. Following are the names of those persons contacted and the agencies with whom they are associated:

Aeronautical Chart and Information Center, U. S. A. F.

Mr. Gerald Johnson
Mr. Adam Mink
Mr. John Neiman

National Aeronautics and Space Administration

Mr. John Oosterhaut
Mr. L. M. Salter

Smithsonian Astrophysical Observatory

Mr. D. Arnold
Mr. R. Downer
Mrs. K. Haramundanis
Mr. R. R. LaCount
Mr. J. Latimer
Mr. R. Martin
Mr. L. McGrath
Mrs. B. Miller
Mr. R. Poland
Mr. J. Rolff
Mr. George Westgate
Mr. R. Wild

Environmental Science Services Administration, Coast and Geodetic Survey

Cdr. E. A. Taylor
Mr. C. Theurer

Environmental Science Services Administration, Institute for Earth Sciences

Dr. Hellmut H. Schmid
Cdr. J. D. Bossler
Mr. Robert Hansen
Mr. Chester Slama

Last, but certainly not least, the author expresses his appreciation to Mrs. Jeanne C. Preston for typing the report.

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1 INTRODUCTION

Geodesy is broadly defined as the science which investigates the form and dimensions of the Earth's surface [Hosmer, 1964, p.1]. Prior to 1958, geodesy was confined to measurements made by classic triangulation and levelling techniques. Attempts have been made to use the "natural" Earth Satellite (the Moon) in observing eclipses of the Sun and occultations of stars to determine ground station positions [Mueller, 1964, pp. 101-126]. However, the relatively large irregular nature of the Moon's surface and infrequency of eclipses made precise occultation and eclipse observations extremely difficult. The advent of man-made satellites, however, made frequent observations of a fairly well-defined point in a stellar background within the capability of modern instrumentation.

The simultaneous photographic observation of a satellite from two or more ground stations (satellite triangulation) is particularly useful in geodetic position determination since the rays formed between the satellite and ground stations uniquely define an unique angular spatial relationship.

Although the "theory" of satellite triangulation is extremely appealing, the practical accomplishment of precise observations demands the utmost precision and discipline from the sciences of astrometry, photogrammetry, and geodesy.

The purpose of this report is to examine the basic theory of stellar triangulation, review the procedures that four agencies currently use in geodetic satellite observations for the National Geodetic Satellite Program, and finally to recommend preprocessing procedures to homogenize the data from the agencies for further geodetic studies. The second section of the report outlines the stellar system and corrections unique to the satellite image in the stellar reference system. The third section outlines the photographic and timing techniques necessary to determine the satellite image position in the stellar background. The fourth section outlines procedures currently used by four agencies observing satellites for the National Geodetic Satellite Program. Finally, the last section recommends methods to preprocess the data from the agencies for geodetic studies.

The reader must regard the report as preliminary, though the procedures outlined have been reviewed by the agencies. The author believes the methods are correct as reported, but misunderstandings about certain procedures may certainly have occurred.

The author assumes this paper will be used by geodesists familiar with the basic terminology used in geodetic astronomy. Conventional "right-handed" rotational matrices are used in the first part of the report. Explanation of the photogrammetric system used requires axes to be permuted and the author explains the permutation process in Section 2.

2 THE STELLAR SYSTEM AND SATELLITE IMAGE CORRECTIONS

The concept of interpolating a satellite direction in reference to a stellar background implies that the observer can determine the relative positions of points within the celestial reference system with respect to the camera station and then determine the parallactic and phase effects that effect the satellite's observed position.

The purpose of this section is to examine the stellar system and develop a rigorous method to update the catalogued stellar positions to observed positions; to outline parallactic and phase corrections that effect the satellite image; and, finally, to outline different ways in which the stellar positions and satellite corrections may be used.

2.1 THE STELLAR SYSTEM

Satellite triangulation implicitly implies that observers at two widely separated points on the Earth's surface simultaneously observe a satellite against different portions of the stellar background. The time of observation must be selected to insure optimum geometric strength between the rays joining the observing stations and the satellite; and, of course, must be during the hours of darkness so the satellite image may be photographed against the stars.

The purpose of this section is to first review the star catalogue systems available; and, second, to outline a method for rigorously updating the star's catalogued position to the observed position.

2.11 Star Catalogue Systems

The basis of a celestial reference system is a star catalogue in which star positions and proper motions are tabulated for a given epoch. Basically, star catalogues can be subdivided into observational catalogues which are based on observations at one observatory (i.e., the Yale and AGK2 Catalogues) and fundamental catalogues which are a combination of selected observational catalogues collated to provide indications of systematic and accidental errors and are referred to a single mean epoch [Mueller and Rockie, 1966, pp. 15-16].

The requirements for an "ideal" star catalogue used in satellite triangulation can be outlined in three basic requirements. First, the catalogue should contain a large number of equally distributed stars of high positional accuracy. Second, the star's positional accuracy (and proper motion accuracy) should be uniquely known with respect to an "absolute" reference system as defined by the catalogue (i.e., if the catalogue does not have internal systematic errors). Finally, the celestial reference system should be used in the determination of rotational time so the position of the system may be related to the terrestrial systems by time [Veis, 1963, pp. 1-10].

At present, the Fourth Fundamental Catalogue (FK4) is generally accepted to be the catalogue of highest positional accuracy [Scott, 1966]. The FK4 is used in the definition of time by decision of the International Astronomical Union in 1961. Unfortunately, the FK4 only contains 1535 stars, therefore, other less accurate catalogues must be considered in satellite triangulation.

The Smithsonian Astrophysical Observatory (SAO) has recently compiled a catalogue of nearly 259,000 stars with an average distribution of nearly six stars per square degree. The SAO Catalogue essentially combines all available catalogues of high positional accuracy and refers the star positions and proper motions to the FK4 reference system (equinox 1950.0) [SAO, 1966, p. xi]. Although the SAO Catalogue is of great benefit in providing a great number of stars referred to a common catalogue system, the basic question of "absolute" positional accuracy is still in doubt. For example, in the SAO Catalogue compilation, a star position was considered to be "duplicated" if, from two different source catalogues, the star positions (at the epoch 1900.0) agreed within ten seconds of arc in right ascension and declination, the star had the same Durchmusterung numbers and the star magnitudes did not differ by more than 3.0. If the star's position was considered to be "duplicated" the more accurate source catalogue position was used [SAO, 1966, p. xiv]. The above example should illustrate that, although stars selected for inclusion in the catalogue had an average standard deviation of $0\overset{''}{.}2$ at the epoch 1900.0, systematic errors may exist (particularly in zones where stars of the highest positional accuracy were not available for comparison) due to uncertainties in the original catalogues. Geodetic investigators should, therefore, use caution in interpreting the tabulated standard deviations of a star's coordinates (or proper motion) in an absolute sense.

The magnitude of systematic errors which may exist in fundamental catalogue systems is one of the foremost problems in fundamental astrometry.

Astronomers have realized for some time that discrepancies exist between fundamental catalogue systems. For example, the fundamental catalogue N-30 of Morgan was formed by compiling 70 recent catalogues with middle epochs of observations between 1920.0 and 1950.0. Although the N-30 cannot be considered as accurate as the FK4, it has aided astrometric investigators uncover systematic errors (particularly in the Southern Hemisphere) in the GC and FK4 systems [Podobed, 1965, p. 221]. Schmid has indicated the magnitude of the errors that exist between the N-30 and FK4 systems will reach 0.9 seconds of arc in right ascension at -60 degrees declination by 1975 [Schmid, 1964, p. 16]. Astrometric surveys are currently in progress to improve the positions and proper motions of stars to the ninth magnitude [Scott, 1963, pp. 8-13]. Preliminary results indicate that the FK4 catalogue will be used as the fundamental reference system in the Northern Hemisphere but may not be adequate in the Southern Hemisphere. Preliminary results also indicate that the mean error of the Boss and N-30 catalogues increase in the Southern Hemisphere and the errors are greater for higher magnitude (fainter) stars [Scott, 1966]. First results of the Northern Hemisphere Survey should be available within the next several years and results of the Southern Hemisphere Survey should be available in the 1970's. The results of both surveys will certainly be a welcome addition in improving our knowledge of the accuracy of catalogues presently available.

Presently, however, we must employ the best catalogue available for satellite triangulation. The most logical choice is the SAO Catalogue since, as mentioned earlier, it has been compiled from the accurate catalogues currently available and referred to the FK4 system. For rigorous plate reduction, as many stars as possible should be used to minimize random errors and provisions should be made for weighting the stars based on the standard deviations tabulated. The geodesist using the catalogue must remember,however, that possible systematic errors may exist in the catalogue. The magnitude of the errors will be larger if fainter stars are used and if the observations are made in the Southern Hemisphere.

2.12 Star Updating Procedures

All catalogue systems are referenced to a heliocentric system referred to a specific mean equator and equinox. All star catalogues (including the SAO Catalogue) represent the geometric direction of the star, at the epoch of the catalogue, conventionally modified by eccentric and secular terms of aberration which will be discussed in Section 2.122 [Explanatory Supplement, 1961, p. 145].

As mentioned in the introduction to this section, stellar triangulation assumes the stellar coordinate system is centered on the observer (topocentric) and the stars appear as seen by an observer at the station.

The procedures outlined in this section fulfill two objectives. First, a rigorous method of star updating to the apparent position is developed which will be useful later in the report to preprocess satellite observations. Second, the method illustrates as clearly as possible the physical significance of astronomical parameters used in other classic updating procedures.

2.121 Proper Motion Corrections

The heliocentric catalogue positions, as mentioned earlier, are referred to the mean epoch of the catalogue. Consequently, it is necessary to update the position of the star in the same mean heliocentric coordinate system to the position where it will be seen (i.e., the epoch of observation). If we let the vector $\underset{\sim}{X}_1$ indicate the rectangular coordinates of the catalogued position we can write

$$\underset{\sim}{X}_1 = \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \begin{pmatrix} \cos\alpha \cos\delta \\ \sin\alpha \cos\delta \\ \sin\delta \end{pmatrix} , \qquad (2.1)$$

where α and δ represent the right ascension and declination of the catalogued position. The coordinates ($\underset{\sim}{X}_2$) corrected for proper motion can then be expressed by

$$\underset{\sim}{X}_2 = \underset{\sim}{X}_1 + \underset{\sim}{U}\,\underset{\sim}{T} , \qquad (2.2)$$

where

$$\underset{\sim}{U} = \begin{pmatrix} \mu_x & \dot{\mu}_x \\ \mu_y & \dot{\mu}_y \\ \mu_z & \dot{\mu}_z \end{pmatrix} \quad \text{and} \quad \underset{\sim}{T} = \begin{pmatrix} T \\ \frac{1}{2}T^2 \end{pmatrix} . \qquad (2.3)$$

The μ_i and $\dot{\mu}_i$ indicate the first and second derivatives of variable indicated in the i subscript if T is measured in units of time consistent with the derivative used. For example, proper motion in the SAO Catalogue is expressed in seconds of time or arc (for right ascensions or declinations, respectively) per year; therefore, T must be expressed in tropical years from the epoch of the catalogue. The number of tropical years since 1950.0 may be computed in the following manner. First, the Julian Day Number (J1) of zero hours Universal Time (UT) for the date of observation may be taken from Table 14.15 of the Explanatory Supplement, or from the table of "Universal and Sidereal Times" in the current American Ephemeris and Nautical Almanac [A. E. N. A., 196X]. The fraction of a day (FD) from zero hours U. T. must be added to J1 to ascertain the Julian Date of Observation (J2). The epoch of the catalogue in Julian Days (J0) must be subtracted from the Julian Date of Observation (J2) to obtain the interval of time (I), in Julian Days, from the catalogue epoch to date. The Julian Date for the epoch of the SAO Catalogue is 2433282.423. The interval of time thus obtained (I) must be divided by 365.24220, the number of days in a tropical year, to obtain T in tropical years [Explanatory Supplement, 1961, p. 30]. Mathematically this may be stated,

$$\begin{aligned} J2 &= J1 + FD , \\ I &= J2 - J0 = J2 - 2433282.423 , \\ T &= I/365.24220 . \end{aligned} \tag{2.4}$$

The foregoing computations, of course, will give the tropical year interval to a much greater precision than necessary for proper motion computations; however, the precise results will be required in Section 2.123. As described in Scott and Hughes the μ_i and $\dot{\mu}_i$ terms are [Scott and Hughes, 1964, p. 368],

$$\begin{aligned} \mu_x &= -\mu_\alpha \sin\alpha \cos\delta - \mu_\delta \cos\alpha \sin\delta \\ \mu_y &= +\mu_\alpha \cos\alpha \cos\delta - \mu_\delta \sin\alpha \sin\delta \\ \mu_z &= \mu_\delta \cos\delta \end{aligned} \quad . \tag{2.5}$$

and

$$\begin{aligned} \dot{\mu}_x &= -X_1 \mu^2 \ , \\ \dot{\mu}_y &= -Y_1 \mu^2 \ , \\ \dot{\mu}_z &= -Z_1 \mu^2 \ , \end{aligned} \tag{2.6}$$

where

$$\mu^2 = \mu_x^2 + \mu_y^2 + \mu_z^2 = \mu_\alpha^2 \cos^2\delta + \mu_\delta^2 \quad . \tag{2.6a}$$

In the above expression, μ_α and μ_δ are the respective annual proper motions of right ascension and declination. Scott and Hughes also include a correction for the change in proper motions due to radial velocity of the star in the second derivative terms; however, the correction is only significant for 31 stars in the FK4 and since a star's radial velocity is not tabulated in the SAO Catalogue, the radial velocity correction has been omitted in this paper.

The terms in the $\underset{\sim}{U}\underset{\sim}{T}$ matrix product must be converted to radians before they are added to the $\underset{\sim}{X_1}$ matrix.

2.122 Annual Aberration Corrections

The cartesian coordinates, corrected for proper motion (X_2), are heliocentric, refer to the equator and equinox of the catalogue, and are corrected to the epoch of observation. The next step is to correct the coordinates for the motion of the Earth about the center of mass of the heliocentric system in space. The effect of secular aberration on the star's position is generally not known and is assumed to be implicitly included in the catalogue positions [Explanatory Supplement, 1961, p. 46].

The correction for annual aberration is presently given by the aberrational day numbers C and D tabulated daily for the year in question in the Nautical Almanac for 0^h Ephemeris Time [A. E. N. A., 196X]. The numbers are currently derived from the true velocity of the Earth in its orbit, referred to the center of mass of the Solar System and to an inertial frame of reference [Explanatory Supplement, 1961, p. 46]. If E_x, E_y and E_z are the components of the Earth's velocity in the inertial X, Y, Z system, and c is the velocity of light, then the aberrational Besselian Day Numbers C and D correspond to $\frac{E_y}{c}$ and $-\frac{E_x}{c}$, respectively. The velocity components $\frac{E_x}{c}$ and $\frac{E_y}{c}$ represent corrections for annual aberration due to the Earth's motion in the inertial reference frame. However, due to the eccentric nature of the Earth's orbit the aberration terms as computed above contain nearly constant terms which depend on the eccentricity ($e_\oplus$) and the longitude of the perihelion of the Earth's orbit ($\overline{\omega}$). By convention, the nearly constant terms (designated the "E" terms of aberration) have been allowed to remain in the catalogued places of stars as mentioned in Section 2.12. If K is the constant of aberration ($20''.47$) and ϵ is the obliquity of the ecliptic, the expression for C and D as described above must be modified to remove the "E" terms by the expression [Porter and Sadler, 1950],

$$
\begin{aligned}
C &= \frac{E_y}{c} - Ke_\oplus \cos\overline{\omega}\cos\epsilon \quad , \\
D &= -\frac{E_x}{c} - Ke_\oplus \sin\overline{\omega} \quad .
\end{aligned}
\qquad (2.7)
$$

Since e, $\bar{\omega}$, and ϵ are a function of the intial epoch of observation of the star concerned (i. e., when the star was observed to determine the catalogue position) a completely rigorous reduction would necessitate removing elliptic aberration at the star's position at the initial epoch and applying new "E" terms for the star's current epoch of observation after precessing the coordinate system to the current epoch. Scott has indicated that, in general, the effect of neglecting this correction is valid except when precessing close polar stars to distant epochs [Scott, 1964]. Table 1 from Scott's article indicates the effect of the changes in elliptic aberration in precessing the position of Polaris from an initial epoch of 1950.0. The effect on coordinates of all other stars in the FK4 is less than that shown in Table 1, consequently the effect will not be considered further.

TABLE 1

THE EFFECT CAUSED BY CHANGES IN ELLIPTIC ABERRATION
IN PRECESSING THE POSITION OF POLARIS

EQUINOX	$\Delta\alpha$	$\Delta\delta$
1960.0	$-0\overset{s}{.}0008$	$0\overset{''}{.}000$
1980.0	$-0\overset{s}{.}0028$	$0\overset{''}{.}000$
2000.0	$-0\overset{s}{.}0052$	$0\overset{''}{.}000$

The aberrational terms (C, D) as mentioned earlier, are tabulated daily for 0^h Ephemeris Time (E. T.). The time of observation will most probably be in Universal Time (U. T.) so it is necessary to add the extrapolated value of the difference between E. T. and U. T. The extrapolated difference between

E. T. and U. T. for 1965.5 is +35 seconds and for 1966.5 is +36 seconds [A. E. N. A., 1966, p. vii]. The values of C and D can then be interpolated from the current Nautical Almanac for the E. T. of observation and will refer to the epoch of observation and the mean equator and equinox at the beginning of the nearest Besselian Year [A. E. N. A., 196X]. Since the values of C and D refer to the mean equator and equinox at the first of the nearest Besselian Year, the values must be precessed to the 1950.0 epoch. The Explanatory Supplement gives the development of and outlines the following formulas for conversion of the C_0 and D_0 terms of equinox 1950.0 to any other equinox t by the following first order expressions [Explanatory Supplement, 1961, p. 160]:

$$
\begin{aligned}
C_t &= C_0 - 0.0002235\,(t - 1950)\,D_0 \quad , \\
D_t &= D_0 + 0.0002656\,(t - 1950)\,C_0 \quad ,
\end{aligned}
\tag{2.8}
$$

where C_t, D_t indicate the day numbers referred to equinox t (for example, 1967). The above first order expressions are accurate to 0".0005 provided t is less than 1980. The values of C and D taken from the Almanac, however, correspond to C_t and D_t and we require C_0 and D_0 (referred to the mean equator and equinox of 1950.0). Solving the above equations for C_0 and D_0 we have

$$
\begin{aligned}
C_0 &= \frac{C_t + 0.0002235\,(t - 1950.0)\,D_t}{1 + 5.93616 \times 10^{-8}\,(t - 1950.0)^2} \quad , \\
D_0 &= \frac{D_t - 0.0002656\,(t - 1950.0)\,C_t}{1 + 5.93616 \times 10^{-8}\,(t - 1950.0)^2} \quad .
\end{aligned}
\tag{2.9}
$$

Since the denominator of the expressions will not vary from one by more than 6×10^{-5} if t is less than 1980 we can write the expression as

$$
\begin{aligned}
C_o &= C_t + 0.0002235\,(t - 1950.0)\,D_t \quad , \\
D_o &= D_t - 0.0002656\,(t - 1950.0)\,C_t \quad .
\end{aligned}
\tag{2.10}
$$

A more rigorous reduction, if desired, can be made by precessing the coordinates by the following matrix expressions [Mueller, in press]:

$$
\begin{pmatrix} -D_o \\ C_o \\ C_o \tan \epsilon_o \end{pmatrix} = \underset{\sim}{R_3}\,(+z)\,\underset{\sim}{R_2}\,(-\theta)\,\underset{\sim}{R_3}\,(+\zeta_o) \begin{pmatrix} -D_t \\ C_t \\ C_t \tan \epsilon_t \end{pmatrix} \quad . \tag{2.11}
$$

The expressions for ζ, θ, and z are given in the next section and must be evaluated for the beginning of the Besselian Year to which the day numbers are referred. The terms ϵ_o and ϵ_t denote the mean obliquity of the ecliptic at the beginning of the Besselian Year at the catalogue and final epochs, respectively. The mean obliquity ϵ_o is given implicitly by the matrix rotations and ϵ_t may be computed from [Explanatory Supplement, 1961],

$$
\begin{aligned}
\epsilon_t = {} & 23^{\circ}\!.452294 - 0^{\circ}\!.0130125T - 0^{\circ}\!.00000164T^2 \\
& + 0^{\circ}\!.000000503T^3 \quad .
\end{aligned}
\tag{2.12}
$$

The value of T in the above expression is the number of Julian Ephemeris Centuries of 36525 ephemeris days since Julian Day 2415020.0 (Epoch 1900 January 0.5 E.T.) to the beginning of the Besselian Year in question. The number of tropical centuries from 1900.0 may also be used for T in the above expression with sufficient accuracy.

The values of $C_\circ$ and $D_\circ \tan \epsilon_\circ$, ascertained above, now refer to the velocity of the Earth with respect to the catalogue equator and equinox (of 1950.0). Consequently, the aberration correction may be expressed as

$$\underset{\sim}{X}_3 = \underset{\sim}{A}_\circ + \underset{\sim}{X}_2 \ , \tag{2.13}$$

where

$$\underset{\sim}{A}_\circ = \begin{pmatrix} -D_\circ \\ C_\circ \\ C_\circ \tan \epsilon_\circ \end{pmatrix} \ . \tag{2.14}$$

$\underset{\sim}{X}_3$ is the cartesian coordinate vector corrected for annual aberration and proper motion in the 1950.0 reference system and the values of $C_\circ$ and $D_\circ$ are expressed in radians.

Authors have recommended procedures for computing C and D for the epoch of observation by computing of the true longitude and radius vector of the Sun using closed expression that are a function of time [Wickens and Jones, 1960]. The author believes, however, that the most rigorous approach is to use the day number values tabulated in the current Nautical Almanac. The tabulated values of C and D are functions of the solar coordinates for 1950.0 which are reduced to the center of mass of the Solar System by a complicated summation process which cannot be conviently expressed in a closed mathematical expression.

2.123 Precession and Nutation Corrections to Date

The star's cartesian coordinates ($\underset{\sim}{X}_3$) corrected for proper motion and annual aberration must now be corrected for the apparent movement of the equator and ecliptic and hence the equinox of the celestial reference system, from epoch to epoch. The movement of the celestial reference system may be described by two general components; general precession and nutation.

The general precession affects the mean pole of the celestial reference system from epoch to epoch and can be described by three angles which are, as shown by Newcomb, a function of time [Explanatory Supplement, 1961, p. 30]. The first angle, $90° - \zeta_o$, is the right ascension of the ascending node of the equator of epoch t on the equator of t_o measured from the equinox of t_o. The second angle, $90° - z$, is the right ascension of the ascending node measured from the equinox of t. Finally, the third angle, θ, is the inclination of the equator of t to the equator of t_o. The three angles ζ, θ, and z are expressed by

$$\begin{aligned} \zeta_o &= (2304''.250 + 1''.396\ T_o)\ T + 0''.302 T^2 + 0''.018 T^3, \\ \theta &= (2004''.682 - 0''.853\ T_o)\ T - 0''.426 T^2 - 0''.042 T^3, \\ z &= \zeta_o + 0''.791 T^2 \quad , \end{aligned} \tag{2.15}$$

where T_o is the interval in tropical centuries from the epoch 1900.0 to the catalogue epoch (0.500 for the SAO Catalogue) and T is the interval in tropical centuries from T_o to date. The precise computation of the interval from 1950.0 to date in tropical years was discussed in Section 2.121, and T may be obtained by dividing the number of tropical years by 100. The application of the following matrix rotations will yield the precessional matrix $\underset{\sim}{P}$ [Mueller, in press],

$$\underset{\sim}{P} = \underset{\sim}{R}_3\,(-z)\;\underset{\sim}{R}_2\,(\theta)\;\underset{\sim}{R}_3\,(-\zeta_o) \quad . \tag{2.16}$$

The elements of the 3 x 3 $\underset{\sim}{P}$ matrix are

$$\underset{\sim}{P} = \begin{pmatrix} P_{xx} & P_{xy} & P_{xz} \\ P_{yx} & P_{yy} & P_{yz} \\ P_{zx} & P_{zy} & P_{zz} \end{pmatrix} \quad , \tag{2.17}$$

where,

$$\begin{aligned}
P_{xx} &= \cos\zeta_o \cos\theta \cos z - \sin\zeta_o \sin z \,, \\
P_{xy} &= -\sin\zeta_o \cos\theta \cos z - \cos\zeta_o \sin z \,, \\
P_{xz} &= -\sin\theta \cos z \,, \\
P_{yx} &= \cos\zeta_o \cos\theta \sin z + \sin\zeta_o \cos z \,, \\
P_{yy} &= -\sin\zeta_o \cos\theta \sin z + \cos\zeta_o \cos z \,, \\
P_{yz} &= -\sin\theta \sin z \,, \\
P_{zx} &= \cos\zeta_o \sin\theta \,, \\
P_{zy} &= -\sin\zeta_o \sin\theta \,, \\
P_{zz} &= \cos\theta \quad .
\end{aligned} \tag{2.18}$$

The star's cartesian coordinate vector ($\underset{\sim}{X}_4$) corrected for precession is then

$$\underset{\sim}{X}_4 = \underset{\sim}{P}\,\underset{\sim}{X}_3 \;. \tag{2.19}$$

The precessed coordinates must now be corrected for nutation at the epoch of observation. The elements of nutation are the nutation in obliquity ($\Delta\epsilon$), the nutation in longitude ($\Delta\psi$), and the mean obliquity of the ecliptic (ϵ_m) for a given epoch. The $\Delta\epsilon$ and $\Delta\psi$ terms are periodic functions depending on the location of the Sun and Moon at the epoch of observation and are currently computed from series expressions developed by Woolard [Woolard, 1953]. The series expressions currently used include 69 terms in $\Delta\psi$, 46 terms in $\Delta\epsilon$, and include all terms with coefficients greater than 0".0002. The coordinates can be computed as outlined in the Explanatory Supplement [Explanatory Supplement, 1961, pp. 43 - 45]. The argument needed for the series is the time interval, in Julian Ephemeris Days and Centuries, from Julian Ephemeris Date (J. E. D.) 2415020.0. The Julian Ephemeris Date for the epoch of observation may be obtained by determining the Julian Date of 0^h U. T. and then adding the fraction of a day since 0^h U. T. including the extrapolated value of the difference between U. T. and E. T. (see Section 2.122). The time interval is then obtained by subtracting 2415020.0 from the J. E. D. for the epoch of observation. A computer program subroutine is currently available at The Ohio State University to perform the $\Delta\epsilon$ and $\Delta\psi$ computation [Allen, 1966]. The mean obliquity of the ecliptic (ϵ_m) may then be computed from equation 2.12 where T for the equinox of date is given in Julian Ephemeris Centuries from J. E. D. 2415020.0 to the epoch of observation. The number of Julian Centuries may be computed by finding the Julian Date of observation of 0^h U. T., adding the time interval from 0^h U. T. converted to a fraction of a day (corrected to E. T. as explained in Section 2.122), subtracting the initial Julian Ephemeris Date (2415020.0) and, finally, dividing by 36525 (the number of Ephemeris Days per Julian Ephemeris Century).

The matrix rotations necessary to apply nutation are, first a rotation into the ecliptic coordinate system ($\underset{\sim}{R}_1(\epsilon_m)$), second a rotation in longitude ($\underset{\sim}{R}_3(-\Delta\psi)$) and, finally, a rotation of coordinates to the true equatorial system ($\underset{\sim}{R}_1(-\epsilon_m-\Delta\epsilon)$).

The matrix resulting from the rotations (N) is the nutation matrix [Mueller, in press]:

$$\underset{\sim}{N} = \underset{\sim}{R_1}\ (-\epsilon_m - \Delta\epsilon)\ \underset{\sim}{R_3}\ (-\Delta\psi)\ \underset{\sim}{R_1}\ (\epsilon_m) \quad . \tag{2.20}$$

If we denote $\underset{\sim}{N}$ as :

$$\underset{\sim}{N} = \begin{pmatrix} N_{xx} & N_{xy} & N_{xz} \\ N_{yx} & N_{yy} & N_{yz} \\ N_{zx} & N_{zy} & N_{zz} \end{pmatrix} \quad . \tag{2.20a}$$

If we let $(-\epsilon_m - \Delta\epsilon) = -\bar{\epsilon}$, the elements of the $\underset{\sim}{N}$ matrix are

$$\begin{aligned}
N_{xx} &= \cos \Delta\psi \ , \\
N_{xy} &= -\sin \Delta\psi \cos \epsilon_m \ , \\
N_{xz} &= -\sin \epsilon_m \ \sin \Delta\psi \ , \\
N_{yx} &= \sin \Delta\psi \cos \bar{\epsilon} \ , \\
N_{yy} &= \cos \bar{\epsilon} \ \cos \epsilon_m \cos \Delta\psi + \sin \bar{\epsilon} \ \sin \epsilon_m \ , \\
N_{yz} &= \cos \bar{\epsilon} \ \sin \epsilon_m \cos \Delta\psi \ - \sin \bar{\epsilon} \ \cos \epsilon_m \ , \\
N_{zx} &= \sin \bar{\epsilon} \ \sin \Delta\psi \ , \\
N_{zy} &= \sin \bar{\epsilon} \ \cos \epsilon_m \cos \Delta\psi - \cos \bar{\epsilon} \ \sin \epsilon_m \ , \\
N_{zz} &= \sin \bar{\epsilon} \ \sin \epsilon \ \cos \Delta\psi + \cos \bar{\epsilon} \cos \epsilon_m \ .
\end{aligned} \tag{2.20b}$$

The maximum values of $\Delta\psi$ and $\Delta\epsilon$ are 19 and 10 seconds of arc, respectively [Explanatory Supplement, 1961, p. 490]. Consequently, we can simplify the above expressions by making the following assumptions:

$$\begin{aligned}
&\cos \Delta\psi \cong 1, \ \sin \Delta\psi \cong \Delta\psi, \\
&\cos \Delta\epsilon \cong 1, \ \sin \Delta\epsilon \cong \Delta\epsilon, \\
&\cos \bar{\epsilon} \cong \cos \epsilon_m, \ \sin \bar{\epsilon} \cong \sin \epsilon_m \ , \\
&\sin \epsilon_m \cos \bar{\epsilon} - \cos \epsilon_m \ \sin \bar{\epsilon} = \sin (\epsilon_m - \bar{\epsilon}) \cong - \Delta\epsilon \ , \\
&\sin \epsilon \ \cos \epsilon_m - \cos \bar{\epsilon} \ \sin \epsilon_m = \sin (\bar{\epsilon} - \epsilon_m) \cong + \Delta\epsilon \ .
\end{aligned} \tag{2.21}$$

The elements of the N matrix then become

$$
\begin{aligned}
N_{xx} &= 1 \ , \\
N_{xy} &= -\Delta\psi \cos \epsilon_m \ , \\
N_{xz} &= -\Delta\psi \sin \epsilon_m \ , \\
N_{yz} &= \ \Delta\psi \cos \epsilon_m \ , \\
N_{yy} &= \cos^2 \epsilon_m + \sin^2 \epsilon_m = 1 \ , \\
N_{yz} &= \cos \bar{\epsilon} \sin \epsilon_m - \sin \bar{\epsilon} \cos \epsilon_m = - \Delta\epsilon \ , \\
N_{zx} &= \Delta\psi \sin \epsilon_m \ , \\
N_{zy} &= \sin \bar{\epsilon} \cos \epsilon_m - \cos \bar{\epsilon} \sin \epsilon_m = \ \Delta\epsilon \ , \\
N_{zz} &= \sin^2 \epsilon_m + \cos^2 \epsilon_m = 1 \ .
\end{aligned}
\qquad (2.22)
$$

The maximum values of the second order terms neglected in the above expressions are in units of 10^{-8} [Scott and Hughes, 1964, p. 370]:

$$
\begin{aligned}
N_{xx} &= 0.39 & \qquad N_{yz} &= 0.14 \\
N_{xy} &= 0.00 & \qquad N_{zx} &= 0.38 \\
N_{xz} &= 0.00 & \qquad N_{zy} &= 0.14 \\
N_{yx} &= 0.17 & \qquad N_{zz} &= 0.17
\end{aligned}
\qquad (2.22a)
$$

$$N_{yy} = 0.44$$

The $\underset{\sim}{N}$ matrix containing only first order terms has a very definite practical advantage. As mentioned in Section 2.122, the Besselian Day Numbers C and D must be taken from a current ephemerides. If we also take the Besselian Day Numbers A, B and E and the fraction of a Besselian Year (τ) from the same tabular system, we can compute values of $\Delta\psi \cos \epsilon$ and $\Delta\psi \sin \epsilon$ without using the long, hence costly, procedure of calculating $\Delta\psi$ and $\Delta\epsilon$ explicitly as outlined earlier. By definition, we can write the day numbers in the following form [Explanatory Supplement, 1961, p. 151]:

$$
\begin{aligned}
A &= n\tau + \sin \epsilon \, \Delta\psi \ , \\
B &= -\Delta\epsilon \ , \\
f &= A \frac{m}{n} + 15 \, E = m\tau + \cos \epsilon \, \Delta\psi \ ,
\end{aligned}
\tag{2.23}
$$

where f is an "independent" day number formed from the Besselian Day Numbers A and E, τ is the fraction of a Besselian Year at which the observation was made and measured from beginning of the nearest Besselian Year. The tabulated value of E must be multiplied by 15 since it is customarily given in seconds of time. The values of m and n are then given, in seconds of arc, by the following expressions [Explanatory Supplement, 1961, p. 38]:

$$
\begin{aligned}
m &= 46''.09905 \ +0''.02790T \ , \\
n &= 20''.0426 \ -0''.0085T \ ,
\end{aligned}
\tag{2.24}
$$

where T is measured in tropical centuries from 1950.0 (see Section 2.121). The values required for the first order nutation matrix can then be expressed in seconds of arc by

$$
\begin{aligned}
\Delta\psi \sin \epsilon &= A - n\tau \ , \\
\Delta\psi \cos \epsilon &= f - m\tau .
\end{aligned}
\tag{2.25}
$$

The above values must be converted to radians before use in the first order nutation matrix. The precessed coordinates ($\underset{\sim}{X}_4$) may then be corrected for nutation by

$$
\underset{\sim}{X}_5 = \underset{\sim}{N} \, \underset{\sim}{X}_4 \quad , \tag{2.26}
$$

where $\underset{\sim}{X}_5$ indicates the coordinate vector corrected for precession, nutation, aberration, and proper motion.

2.124 Parallax Corrections

To correct the position $\underset{\sim}{X}_5$ obtained in the previous section to the geocenter, the coordinates must be corrected for parallax. Scott gives mathematical formulae for parallax corrections to be applied to the $\underset{\sim}{X}_5$ coordinates [Scott, 1964, p. 370]. However, the SAO Catalogue does not tabulate the parallax corrections for star positions.

Annual parallaxes have been measured for many stars; however, only about 700 stars have parallaxes large enough (about $0''.05$ or more) to be measured with a precision of 10%. Of the approximately 700 stars with reliable parallaxes, most are invisible to the naked eye. The largest annual parallax that has been detected is about $0''.76$ for a system of three stars in the Southern Hemisphere [Abell, 1964, pp. 327-328].

Since the annual parallaxes of a star are very small, no appreciable error should be introduced by ignoring the correction.

The geocentric parallax due to displacement of the topocentric position from the geocenter is, of course, also negligable.

2.125 Cartesian Coordinate Conversion

The cartesian vector $\underset{\sim}{X}_5$ represents a star's apparent position corrected for proper motion, aberration, precession, nutation and (as stipulated in the previous section) parallax. The next correction, diurnal aberration, however, is most readily applied to the spherical coordinates of the star position.

The conversion from the cartesian coordinate system to the spherical coordinate system may be accomplished as follows:

$$\underset{\sim}{X}_5 = \begin{pmatrix} \cos\delta_5 & \cos\alpha_5 \\ \cos\delta_5 & \sin\alpha_5 \\ \sin\delta_5 & \end{pmatrix} = \begin{pmatrix} x_5 \\ y_5 \\ z_5 \end{pmatrix} \quad . \tag{2.27}$$

Thus the spherical coordinates may be determined from

$$\begin{aligned} \delta_5 &= \arcsin z_5 = \arctan \frac{z_5}{\sqrt{x_5^2 + y_5^2}} \quad , \\ \alpha_5 &= \arctan \frac{y_5}{x_5} \quad . \end{aligned} \tag{2.28}$$

The sign of α_5 may be ascertained from the numerator and denominator of the arctangent (the cosine of δ_5 will always be positive).

2.126 Diurnal Aberration Corrections

The geocentric spherical coordinates must be corrected for aberration due to rotation of the observer's position about the center of the Earth. The diurnal aberrational displacement is given by [Explanatory Supplement, 1961, p. 49] ,

$$\begin{aligned} \Delta\delta &= \delta_6 - \delta_5 = 0''.320\, \rho \cos\varphi' \cos h \sec\delta_5 \quad , \\ \Delta\alpha &= \alpha_6 - \alpha_5 = 0^s.0213\, \rho \cos\varphi' \sin h \sin\delta_5 \; , \end{aligned} \tag{2.29}$$

where ρ is the radius of the Earth at the observer's station divided by the equatorial radius, φ' is the geocentric latitude, h is the hour angle of the star, and δ_5 is the apparent declination. Since the corrections are quite small, we can write with sufficient accuracy

$$\begin{aligned} \rho &= 1 \quad , \\ \varphi' &= \varphi \quad , \end{aligned} \tag{2.30}$$

where φ is the geodetic latitude. The hour angle can then be formed from

$$h = \text{LAST} - \alpha_5 \quad , \tag{2.31}$$

where the Local Apparent Sidereal Time (LAST) may be computed from

$$\begin{aligned} \text{LAST} &= \text{U. T.} + \lambda + 9\overset{s}{.}8565 \times \text{U. T. (hrs)} \\ &+ \text{G. A. S. T.} + \Delta \text{ Eq. E} \quad , \end{aligned} \tag{2.32}$$

where U. T. is the Universal Time of Observation, λ the longitude (positive when east), $9\overset{s}{.}8565$ converts the U. T. interval from 0^h U. T. to a mean sidereal interval, G. A. S. T. is the Greenwich Apparent Sidereal Time at 0^hU. T., and Δ Eq. E. is the interpolated difference of the equation of the equinoxes for the G. A. S. T. epoch at 0^hU. T. preceding and following the epoch of observation. Since the corrections are small, the precise type of U. T. used and the precise longitude used are not critical and Δ Eq. E may be neglected in computing the diurnal aberration correction.

2.127 Stellar Refraction Corrections

The Earth's atmosphere tends to "bend" an incoming light ray from a star, consequently, the star's observed position will be apparently displaced toward the observer's zenith. From a mathematical standpoint the amount of displacement of the ray can be expressed as a function of the unrefracted zenith distance of the star, the index of refraction of the atmosphere, and the thickness of the atmosphere. The index of refraction of the atmosphere is, of course, a function of the wavelength of the incoming light and the pressure and temperature of the atmosphere. Many formulas have been devised for various assumed models of the Earth's atmosphere [Baldini, 1963; Schmid, 1963]. One of the strongest models, that is currently used in satellite triangulation, is the model devised by Garfinkel in 1944. Brown has shown that the Garfinkel method is capable of providing accuracies greater than one second of arc for zenith distance up to 90° providing the first four coefficients used in the Garfinkel expansion can be determined accurately [Brown, 1957, pp. 40-42].

The basic Garfinkel model is of the form

$$\Delta Z = \eta_1 \tan \bar{\theta} + \eta_2 \tan^3 \bar{\theta} + \eta_3 \tan^5 \bar{\theta} + \dots \quad , \tag{2.33}$$

where ΔZ indicates the unrefracted zenith distance minus the refracted zenith distance $(Z_o - Z_R)$, θ indicates an auxiliary angle described below, and η_1, η_2 ... are coefficients dependent on the physical state of the atmosphere. The auxiliary angle $\bar{\theta}$ may be computed from

$$\cot 2\bar{\theta} = \gamma_o \cot Z_R \quad , \tag{2.34}$$

where Z_R is the refracted zenith distance, and γ_o is an atmospheric constant equal to

$$\gamma_o = 8.1578 \sqrt{273/T} \quad , \tag{2.35}$$

where T is the temperature in degrees Kelvin at the observing site. The coefficients η_i used in the Garfinkel expansion may either be computed explicitily or taken from tables given in [Garfinkel, 1944, p. 78]. Precise techniques currently used will be given in the agencies procedures later in this report.

The zenith distance argument used in the Garfinkel expression is the refracted zenith distance (Z_R). If we want the refracted zenith distance, but are given the unrefracted zenith distance (Z_o), it is necessary to reiterate the Garfinkel expressions (equations 2.33 and 2.34) using the Z_R as a first approximation.

2.2 SATELLITE IMAGE CORRECTIONS

In the previous section we considered the factors necessary to correct the catalogued star positions to observed topocentric positions. However, the actual satellite position cannot be considered to be the satellite position observed on the photographic plate referred to the stellar background. As shown in Section 3 the plate reduction process essentially "interpolates" the unknown satellite image from the known stellar images on the photographic plate. Therefore, the satellite image coordinates from the plate reduction may be interpreted to be the coordinates of a fictitious star at an infinite distance from the observer and fixed on the celestial sphere. The satellite, of course, is not at an infinite distance from the observer and is moving relative to the observer in an orbital path. Therefore, the satellite position as determined from this type of plate reduction differs from the actual satellite position.

The "differences" in the observed and actual satellite position are due primarily to four factors. First, the satellite is moving with a definite velocity with respect to the observer; therefore, the satellite will be displaced or aberrated by an amount dependent on the satellite's velocity relative to the observer. Second, the satellite is a finite distance from the Earth, consequently, astronomic refraction corrections applied to the observed coordinates of the satellite position obtained from a photographic plate will not represent the actual satellite position, but must be corrected for differential refraction as shown in Figure 1. Third, if the satellite is passive, its geometric center will not coincide with the observed center since only a portion of the satellite will be illuminated by the Sun. Finally, the irregular nature (or shimmer) of the Earth's atmosphere causes the satellite image to be displaced in an irregular manner on the photographic plate.

Each of the above four factors affecting the satellite image are discussed in this section. However, before discussing the correction it is important to note that all of the corrections are functions of the distance of the satellite from the observer (except the satellite aberration correction for an active satellite and the satellite shimmer effect). In order to apply the corrections, we must either use orbital data to determine the satellite range, or compute a "nearly" simultaneous intersection from ground stations (assuming the ground station coordinates are correct) to determine the satellite range. The implications of this decision will be discussed further in Section 5.1.

2.21 Satellite Aberration Corrections

Basically the correction for aberration due to the relative velocity between observer and satellite may be applied either by correcting the satellite's coordinates, or by applying a light time correction to correct the recorded time at the station in the case of a passive satellite [Veis, 1960, pp. 115-117]. The time of flash for an active satellite refers to the time the light is emitted from the satellite and is antedated with respect to the time that would be measured at the plate and no aberration correction is necessary.

The author believes that the most logical way of applying the correction is to antedate the measured satellite time in the case of a passive satellite, or to accept the time of flash as the epoch of satellite observation in the case of an active satellite. Basically, the light time correction can be justified by three advantages. First, the satellite position correction assumes that the rate of change of the satellite's right ascension and declination are known, but the light time correction does not. For example, the following formulas are used [Veis, 1960, p. 116]:

$$\Delta\alpha = \frac{d\alpha_s}{dt} \quad \frac{r_s}{c \cos \delta_s} \quad ,$$

$$\Delta\delta = \frac{d\delta_s}{dt} \quad \frac{r_s}{c} \quad , \tag{2.36}$$

where $\Delta\alpha$ and $\Delta\delta$ are the corrections to the satellite's right ascension (α_s), and declination (δ_s). The $\frac{d\alpha_s}{dt}$ and $\frac{d\delta_s}{dt}$ represent the rate of change of α_s and δ_s with respect to time, r_s is the range to the satellite, and c is the velocity of light. Unless measured during plate reduction, the rates of change of α_s and δ_s will not be known and orbital theory will have to be used to determine the coordinates' rates of change. Second, if the time is not antedated to the epoch at which the light left the satellite, a simultaneous event will be nearly impossible. For example, if a light is reflected from a satellite at time t_s it will reach station A at time $t_s + \frac{r_A}{c}$ and station B at time $t_s + \frac{r_B}{c}$, where r_A and r_B are the satellite-station distances, and c is the velocity of light.

Finally, a flashing light satellite automatically gives the time of satellite flash and no time correction is necessary.

The correction then to a passive satellite's recorded time should be

$$t_s = t - \frac{r}{c} , \tag{2.37}$$

where t is the time of satellite observation at the station, t_s is the time at the satellite, r is the range to the satellite and c is the velocity of light. If r is given in meters, the value of c should be 299,792,500 meters/sec [Mechtly, 1964, p. 5].

Differentiating the above equation we find that

$$dt_s = \frac{dr}{c} . \tag{2.38}$$

Consequently, r should be determined to ±30000 meters to avoid errors in t_s of ±0.1 millisecond.

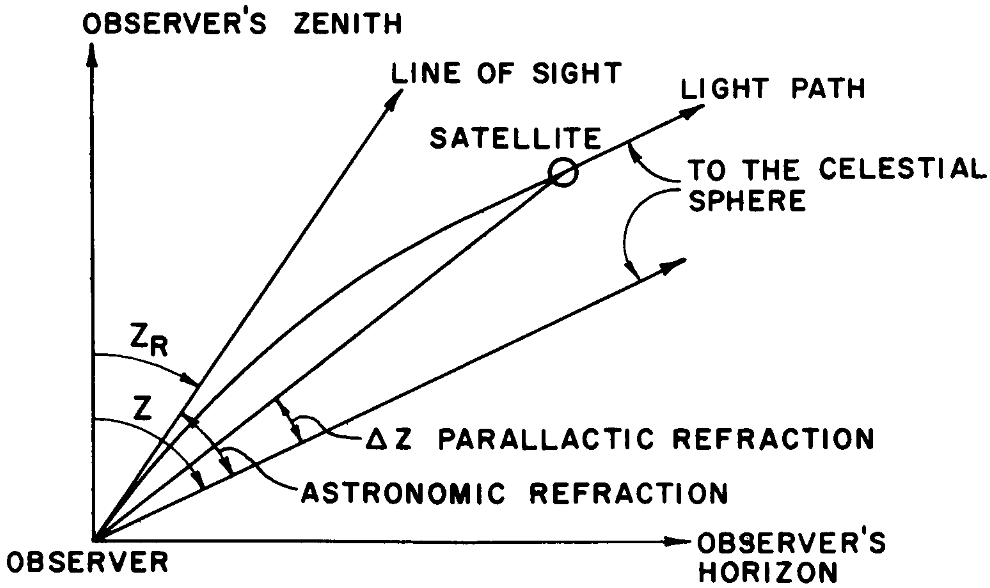
The magnitude of the correction $-r/c$ is, of course, directly dependent on the satellite's range.

2.22 Satellite Refraction Corrections

As mentioned earlier, the satellite image will be "displaced" from its observed position on the celestial sphere since the satellite is a finite distance from the observer. The difference between the apparent position on the celestial sphere and the actual satellite position is defined as the parallactic refraction (ΔZ) of the satellite position as shown in Figure 1. In the figure, Z indicates the zenith distance of the unrefracted ray to a star at an infinite distance from the observer, Z_R indicates the refracted or observed ray, $Z-Z_R$

FIGURE 1

ASTRONOMIC AND PARALLACTIC REFRACTION



is the astronomic refraction, and ΔZ indicates the parallactic refraction.

A number of formulas have been developed to express the parallactic refraction for an object at a finite distance from the observer [Brown, 1957; Baldini, 1963; Schmid, 1963]. The formulas generally are functions of the meteorological parameters of the observing station (or related to the astronomic refraction $Z-Z_R$ as shown in Figure 1), the range to the satellite (or the satellite and observer's distance from the center of the Earth), and the zenith distance.

The specific formulas currently used have been developed by the individual investigators and, therefore, will be included in the section dealing with the individual agencies'procedures later in this report. As an example, however, of the magnitude of the ΔZ correction the formula currently used by the Environmental Science Services Administration is given below [Schmid, 1963, p. 9]:

$$\Delta Z = \frac{2.330 \tan Z_R}{r \cos Z_R} \left(\frac{206265}{1+\beta t}\right) \frac{P_S}{P_O} , \qquad (2.39)$$

where r is the range of the satellite, β is a constant equal to 0.003665, t is the temperature at the station in degrees centigrade, P_S is the barometric pressure at the station in mm of Hg., and P_O is the standard barometric pressure (760 mm Hg.). The formula, as explained in Schmid's article, assumes a flat Earth geometry with the observer at the Earth's surface; therefore, the formula disagrees with more rigorous formulas also developed by Schmid at large zenith distances [Schmid, 1963]. However, the magnitude of disagreement with the most accurate formula given in the paper is only 0.01 seconds of arc at a zenith

TABLE 2

PARALLACTIC REFRACTION, ΔZ (in seconds of arc)

Z_R	r = 100 km	r = 300 km	r = 500 km
0°	0	0	0
15°	1.334 (1.243)	0.445 (0.415)	0.267 (0.249)
30°	3.204 (2.985)	1.068 (0.995)	0.641 (0.597)
45°	6.796 (6.332)	2.266 (2.111)	1.360 (1.266)
60°	16.648 (15.551)	5.550 (5.171)	3.330 (3.103)

distance of 75°. Table 2 illustrates the magnitude of ΔZ in seconds of arc computed with the formula 2.39 for various refraction zenith distances (Z_R) and satellite ranges (r). The ΔZ values without parentheses were computed for $t = 0°C$ and $P_s = 760$ mm, the ΔZ values in parentheses were computed with $t = 10°C$ and $P_s = 760$ mm. The differences of the values computed at a 10°C temperature difference indicate the importance of knowing the temperature at the observing station.

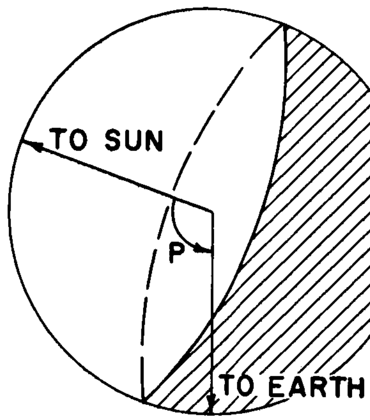
2.23 Passive Satellite Corrections

The portion of a large sun-illuminated satellite facing an observer continuously changes with time in the same manner and for the same reason the Moon goes through its phases. As a result, the center of the continuously changing sunlit image facing the observer will, in most cases, not coincide with the geometrical center of the satellite as shown in Figure 2.

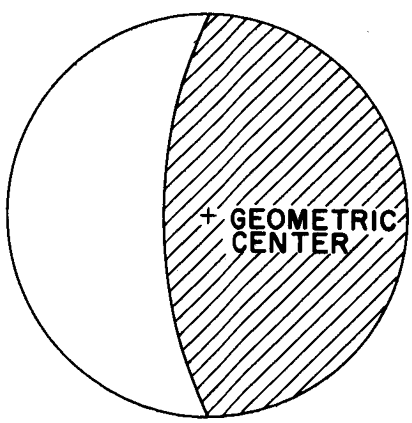
Corrections to be applied to the satellite's right ascension and declination due to the phase of the satellite have been developed by Erwin Schmid at the

FIGURE 2

PASSIVE SATELLITE PHASE



SPACE VIEW



EARTH VIEW

Environmental Science Services Adminstration. The derivation of the formulas are quite lengthy, consequently, only the results are given in this section. Basically, two cases are considered.

The first case involves a spherical satellite which, due to physical characteristics of its surface, diffuses light in all directions from the illuminated surface. In this case, the corrections to be applied to the observed α_S and δ_S of the satellite are

$$\Delta\alpha = \frac{R_S (\cos P - 1)}{2r \sin P \cos \delta_S} \cos \delta_{\odot} \sin (\alpha_{\odot} - \delta_S) \quad ,$$

$$\Delta\delta = \frac{-R_S (\cos P - 1)}{2r \sin P \sin \delta_S} [\cos \delta_{\odot} \cos (\alpha_{\odot} - \alpha_S) + \cos P \cos \delta_S] \quad , \tag{2.40}$$

where r is the distance from the observer to the satellite, $\delta_{\odot}$ and $\alpha_{\odot}$ are the Sun's declination and right ascension interpolated from the Sun's ephemeris for the time of observation, R_S is the radius of the satellite, r is the distance to the satellite, and P is the cosine of the negative dot product of the two vectors given below:

$$\cos P = -\vec{m} \cdot \vec{1} \quad , \tag{2.41}$$

and

$$\begin{aligned} m_1 &= \cos \delta_S \sin \alpha_S & 1_1 &= \cos \delta_{\odot} \sin \alpha_{\odot} \\ m_2 &= \cos \delta_S \cos \alpha_S & 1_2 &= \cos \delta_{\odot} \cos \alpha_{\odot} \\ m_3 &= \sin \delta_S & 1_3 &= \sin \delta_{\odot} \end{aligned} \quad ,$$

or

$$\cos P = -(1_1 m_1 + 1_2 m_2 + 1_3 m_3) \quad . \qquad 2.41b)$$

The above equations apply to a diffusive satellite; however, if the light is not diffused, but perfectly reflective, the equations become

$$\Delta\alpha = \frac{R_s}{r \sin P \cos \delta_s} \left[\frac{1-\cos P}{2}\right]^{\frac{1}{2}} \cos \delta_{\odot} \sin (\alpha_{\odot} - \alpha_s) \ ,$$

(2.42)

$$\Delta\delta = \frac{-R_s}{r \sin P \sin \delta_s} \left[\frac{1-\cos P}{2}\right]^{\frac{1}{2}} [\cos \delta \cos(\alpha_{\odot} - \alpha_s) + \cos P \cos \delta_s] \ .$$

The maximum values of the above expressions can be estimated from Figure 2. The worst case will occur when the satellite is perfectly reflective and the angle P is nearly 90°. In this case, the geometrical center will be "shifted" to nearly $\frac{R_s}{2}$ and the error in radians will be $\frac{R_s}{(2r)}$. Thus, for this extreme example, the measured center of a satellite with a 20 meter radius may appear "shifted" from the geometric center by 20''.63, 4''.13, or 2''.06 at respective distances of 100 km, 500 km, and 1000 km from the observing station.

2.24 Satellite Shimmer Corrections

A final factor effecting the observed satellite position is due to refraction anomalies resulting from atmospheric turbulence [Brown, 1957, p. 50]. The precise nature of the shimmer effect is not well-known, consequently, analytical procedures to eliminate or even reduce the effect are not available. Brown reports that the shimmer effect is larger in magnitude in cold or polar air and smaller in warm tropical air [Brown, 1957, p. 50]. Schmid indicates that for a 200 mm focal length camera the shimmer effect can account for a position error of ±2 seconds of arc based on time series analysis [Schmid, 1964b, p. 20].

As mentioned earlier, there is no reliable way of explicitly compensating for the shimmer effect. If a series of individual satellite images are avaible from a given plate, a time series polynomial may be "fitted" to minimize random errors. However, at present this technique is applicable only in passive satellite reductions since active satellites generally produce only a few (4 to 7) images during each flash sequence.

2.3 USING THE STELLAR AND SATELLITE IMAGE CORRECTIONS

The type of star positions used in the plate reductions, of course, define the coordinate system in which the satellite position is observed. Theoretically, any coordinate system may be used provided the star positions are corrected for proper motion to the epoch of observation, and corrections for the position and motion of the observer in the heliocentric system (i. e., annual and diurnal aberration corrections) are applied (assuming parallax is negligible). Presently, all agencies in effect use the observed stellar positions except the Smithsonian Astrophysical Observatory (SAO) which uses the mean stellar positions of 1950.0. Consequently, this section will only consider these two alternatives.

Agencies using advanced plate reduction procedures (outlined in Section 3) require that the observed place of the stars are used. The observed stellar position includes all of the explicit corrections outlined in Section 2.1. The satellite's position will then be interpolated from the observed star positions during the plate reduction process and the satellite coordinates will represent the observed position of the satellite. If we then correct the satellite's observed coordinates for atmospheric refraction (i.e., astronomic refraction minus parallactic refraction), parallactic aberration, passive satellite phase correction (if applicable), and shimmer (if possible), we will obtain the true topocentric geometric position of the satellite referred to the true equator and equinox of the epoch of observation. This type of position is a most desirable result since the true topocentric system can be related to an Earth fixed system by applying transformations for apparent sidereal time and polar motion. If the vector $\underset{\sim}{X}_S^s$ denotes the satellite's actual topocentric position, the conversion to the mean terrestrial system is given by [Mueller, in press],

$$\underset{\sim}{X}_{mT} = \underset{\sim}{R}_2(-x)\ \underset{\sim}{R}_1(-y)\ \underset{\sim}{R}_3\ (\theta)\ X_S^s \quad , \tag{2.43}$$

where $\underset{\sim}{X}_{mT}$ is the vector describing the cartesian coordinates in the mean terrestrial system, defined by the average North Pole (defined by the International Polar Motion Service) and the mean astronomical meridian of Greenwich (as defined by the Bureau International de l'Heure), x and y are the spherical coordinates of the true celestial pole with respect to the mean terrestrial pole (x is measured positive towards Greenwich, y is measured positive towards 90° west longitude), and θ is the Greenwich apparent sidereal time of observation. As shown by Preuss,the x and y coordinates of the instantaneous pole computed as differential terms; consequently, the above expression yields [Mueller, in press],

$$\underset{\sim}{X}_{mT} = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & -y \\ -x & y & 1 \end{pmatrix} \underset{\sim}{R}_3(\theta)\, \underset{\sim}{X}_5^s \quad . \tag{2.44}$$

Veis also outlines techniques for relating the mean terrestrial system to a given geodetic datum. Since the topocentric satellite direction may be mathematically related to a given geodetic datum, it provides an excellent "system" of coordinates for geodetic adjustment of satellite observations [Veis, 1963, pp. 1-10].

The mean positions of 1950.0, presently used by the SAO, also provide a reference system which can be used. The catalogued (1950.0) coordinates must be corrected for proper motion to the epoch of observation as outlined in Section 2.121. As mentioned previously, however, the stars (and satellite image) recorded on the plate will be in the observed position at the epoch of observation. Therefore, the mean positions of the star images relative to each other will not agree with their relative positions on the plate due to differential precession, differential nutation, annual and diurnal aberration, and astronomic refraction.

From a practical standpoint, the differential quantities may be treated in two ways. First, differential corrections as described in the Explanatory Supplement may be explicitly applied [Explanatory Supplement, 1961, Chapter 2]. Or the first order differential effects may be assumed to be corrected by the constants used in the plate reduction process [Smart, 1962, pp. 288-297].

The astronomic refraction correction may also be treated in two ways. First, the astronomic refraction may be applied explicitly to each star position in the 1950.0 mean coordinate system, and then the satellite position may be corrected by removing the atmospheric refraction (the astronomic refraction minus the parallactic refraction). Or, second, one may assume that the refraction varies linearly over a small area of the plate used in plate reduction, provided the zenith distance is not too large. In this case, the mean star coordinates corrected for proper motion may be used directly in the plate reduction process and the satellite image must be corrected for parallactic refraction only.

In any case, the interpolated satellite image must be also corrected for annual aberration, diurnal aberration, and parallactic aberration to obtain the mean position of the satellite referred to the mean equator and equinox of 1950.0. The true direction to the satellite referred to the true equator and equinox of the epoch of observation may be then obtained by applying additional corrections for precession and nutation as outlined in Section 2.123.

3 PLATE REDUCTION AND TIMING PROCEDURES

3.1 INTRODUCTION

Before considering specific procedures currently used by agencies observing satellites for geodetic purposes, it is necessary to review basic concepts of the photogrammetric stellar plate reduction and timing procedures.

Before considering the procedures, however, we must examine the various methods in which, theoretically, the camera at the observing station may be used. Basically, four alternatives are possible with some cameras.

The first alternative is to allow the camera to remain stationary during the star-satellite exposure. In this case, the stars will produce trails on the photograph plate as the Earth rotates and a shutter system must be provided to interrupt or "chop" the stellar trails at known epochs to produce measurable images. If the observed satellite is active the shutter may then be opened during the satellite flash period. After the active satellite passes, the shutter may again be activated to "chop" the stellar images. If a passive satellite is observed, the same procedure may be used, except the satellite image (which will also appear as a trail on the plate) must be "chopped" at known time intervals.

The second alternative is to drive an equatorially mounted camera system at a sidereal rate. In this case, the stars will appear as point images and an active satellite will appear to move through the star field at a rate dependent on the satellite's velocity relative to the star background. A passive satellite moving through the stellar field must then be "chopped" to produce point images at known epochs. The sidereally driven camera method implies that highly precise mechanical systems are available to move the camera precisely at a sidereal rate. Any "fluctuations" in the drive system will produce undesirable image shifts [Brown, 1964, p. 109]. The star images' physical sizes on the plate will also be increased since the exposure time will extend over the entire time the satellite is being photographed. The increased size of the brighter star images may present difficulties in making accurate plate measurements since the center of the enlarged image must be estimated. Brown outlines in detail thirteen relative disadvantages (including the two given above) of using the sidereally driven camera [Brown, 1964, pp. 109-111]. However, one advantage of the sidereally driven camera is that, when observing active satellites, shutters and associated precise shutter timing equipment are not necessary (since the time of exposure can be readily determined from the time of satellite flash).

The third alternative is to drive the camera at a predicted satellite rate. As Mueller points out, the satellite will theoretically appear as a point image, but in most cases the satellite rate is not known accurately [Mueller, 1964, p. 248]. Therefore, in addition to possible imperfections in drive mechanisms, inaccurate orbital predictions may also cause the satellite image to be blurred. The method is also inadequate for active satellites since the flashes will appear superimposed on each other. The only advantage to the method is that faint satellites may be observed for non-geodetic purposes.

The last alternative is a combination of the orbital and stationary techniques. The camera first makes a regular exposure while tracking the satellite, then the camera is fixed and a second stationary exposure is made. This method is, of course, subject to all of the inaccuracies mentioned in the third alternative; however, the method is useful in roughly identifying a faint satellite in a cluster of faint stars [Mueller, 1964, p. 248].

The mode of operation used in practice depends on the type of camera used, the agencies' procedures, and the timing system associated with the camera.

3.2 PLATE REDUCTION PROCEDURES

Basically, plate reduction involves correlating known and unknown points in object space (i.e., the star and satellite images) with measurable images on a photographic plate (image space). A schematic mathematical statement of the problem may be written [Schmid, 1959, p. 10],

$$\underset{\sim}{U} = \varphi(\underset{\sim}{0}, \underset{\sim}{x}) \quad . \tag{3.1}$$

The above equation schematically represents the general problem of photogrammetry by indicating a functional relationship exists between object space $(\underset{\sim}{U})$, image space $(\underset{\sim}{x})$, and a set of orientation parameters $(\underset{\sim}{0})$.

The purpose of this section is to logically develop the schematic relationship mathematically implied in the above equation, and to show a simplified version of the general relationship which may, and presently is, being used. Since the author's purpose is not to develop a "new" plate reduction process, the development given is limited to the main concepts involved and procedures currently used. For a more complete description of the advanced method of plate reductions, the reader is referred to the development of the method given by Schmid and Brown [Schmid, 1959; Brown, 1957; Brown, 1964].

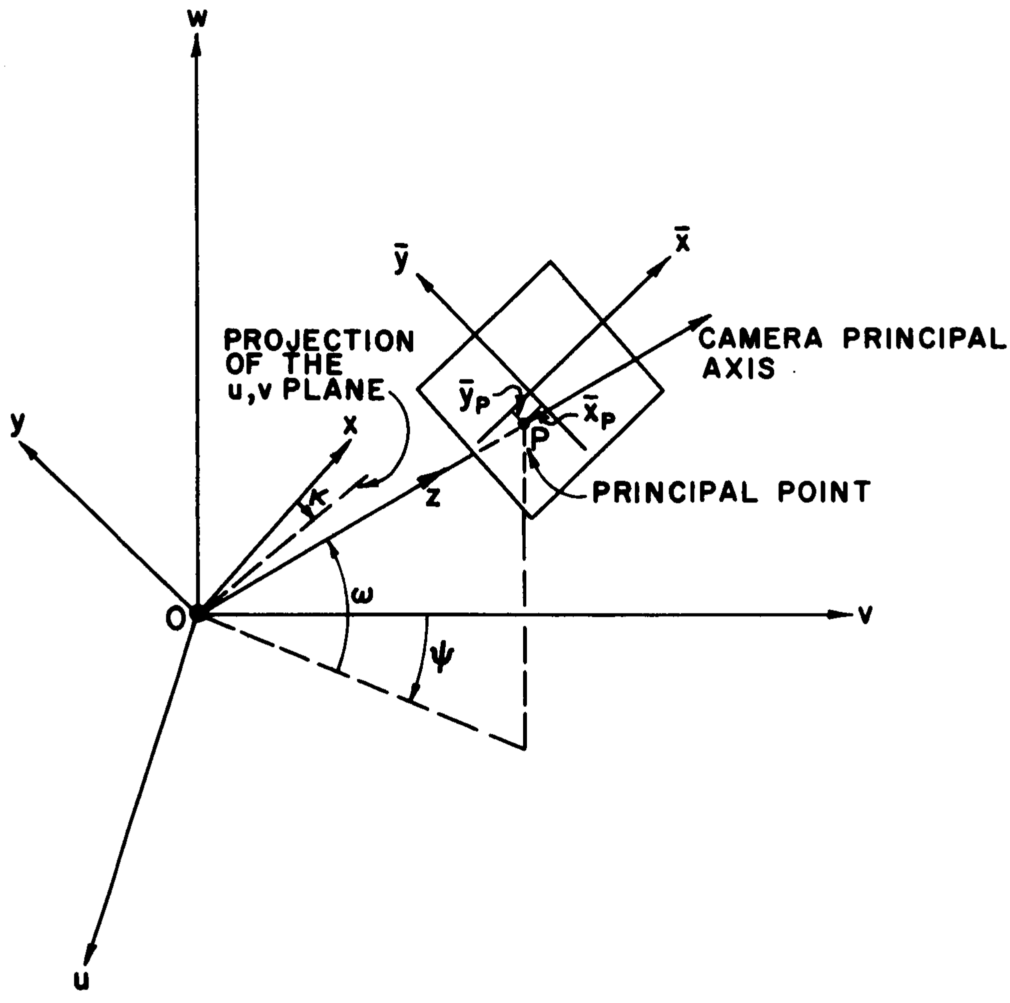
3.21 Central Projection Theory

In the following discussion the author develops the ideal functional relationships implied in the general problem of photogrammetry. Before developing the concepts of the theory, the author must emphasize that the advanced plate reduction techniques are based on physical relationships existing between the object and image space systems; however, the simplified case outlined later does not require physical interpretation.

Figure 3 illustrates the relationship between a topocentric stellar system (u, v, w) as developed in the last chapter, and a camera plate system $(\bar{x}, \bar{y}, \bar{z})$. The w axis in the figure defines the astronomic zenith of the observer, the axis v is oriented to the observer's astronomic north and, consequently, the v, w plane is in the observer's astronomic meridian (free of polar motion). The author must note that the choice of the u, v, w coordinate system can be arbitrary to a certain extent. For example, a different coordinate system is used in the Manual of Photogrammetry [American Society of Photogrammetry, 1966, pp. 183-193]. In the plate coordinate system the $\bar{x}$, $\bar{y}$ plane is defined by the emulsion of the photographic diapositive and the $\bar{z}$ axis is perpendicular to the $\bar{x}$, $\bar{y}$ plane. The origin of the $\bar{x}$, $\bar{y}$, $\bar{z}$ system is arbitrarily determined

FIGURE 3

THE TOPOCENTRIC AZIMUTH -- ALTITUDE AND CAMERA PLATE COORDINATE SYSTEMS



by fiducial marks or other means. The origin of the x, y, z coordinate system is at the projection center, and the x and y axes are parallel to the $\bar{x}$ and $\bar{y}$ axes, respectively.

The point at which a ray passing through the center of projection perpendicularly intersects the $\bar{x}$, $\bar{y}$ plane is called the principal point of the photograph with coordinates $\bar{x}_p$, $\bar{y}_p$. The distance to the projection center from the principal point is the camera's principal distance (c) and the ray passing through the principal point and projection center is the principal axis (of the camera).

If we define the center of the u, v, w topocentric system to be at the center of projection of the camera system as shown in Figure 3, then we can assume that, ideally, all rays passing through the projection center will be straight lines. Thus an undistorted central projection will result from the stellar system to the camera image system. The orientation 'ements of the camera principal axis are, as shown in Figure 3, the astronomic azimuth (ψ) positive when measured from the North (v) to the projection of the camera principal axis on the u, v plane to the East (u), the astronomic elevation (ω) positive when measured from the v, u plane to the camera principal axis toward the zenith (w), and the swing of the camera ($\varkappa$) positive when measured from the x axis of the projection centered system to the u, v plane in the direction of the y axis.

The relationships between the projection center coordinates (x, y, z) for an arbitrary stellar image, the diapositive coordinates $(\bar{x}, \bar{y})$ and principal point $(\bar{x}_p, \bar{y}_p, c)$

are

$$
\begin{aligned}
x &= \bar{x} - \bar{x}_p , \\
y &= \bar{y} - \bar{y}_p , \\
z &= c \quad .
\end{aligned}
\tag{3.2}
$$

If the direction cosines of the stellar image (u, v, w) are known we can relate the photograph plate and topocentric stellar coordinate systems; however, several comments are necessary before proceeding.

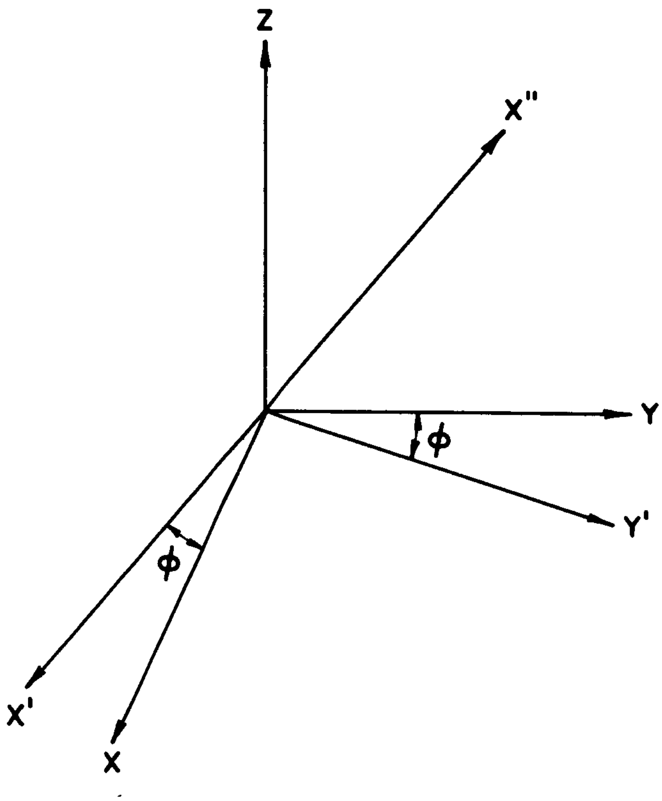
In Section 2 we used conventional rotational matrices in a right-handed system with a positive angle measured in from the X to Y axes in the X, Y plane, and with a positive angle measured from Y to Z in a plane defined by Z intersecting the X, Y plane. In Section 2 we also used straight-forward "right-handed" coordinate systems. We must now consider how to modify the specific rotations to change the direction of a positive axis. For example, as shown in Figure 4, an arbitrary X, Y, Z system can be permuted in one or more axes to provide great flexibility. The permutation matrices for the three axes are

$$
\begin{aligned}
\underset{\sim}{P}_1 &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \\
\underset{\sim}{P}_2 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \\
\underset{\sim}{P}_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} .
\end{aligned}
\tag{3.3}
$$

Thus, as shown in the figure, $\underset{\sim}{P}_1 \underset{\sim}{R}_3 (-\varphi)$ will rotate the X, Y, Z coordinates

FIGURE 4

ROTATION AND PERMUTATION OF
A COORDINATE SYSTEM



$$\begin{pmatrix} X'' \\ Y' \\ Z \end{pmatrix} = \underset{\sim}{P_1}\ \underset{\sim}{R_3}\ (-\theta) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

to X', Y', Z and reverse the positive X' axis to X''. The permuted rotation is then

$$\underset{\sim}{P}_1\,\underset{\sim}{R}_3(-\varphi) = \begin{pmatrix} -\cos\varphi & +\sin\varphi & 0 \\ +\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad . \tag{3.4}$$

In all cases, the matrix rotations are positive if the (X) axis is rotated into the 2 (Y) axis, the 2(Y) axis into the three axis (Z), and the three axis (Z) into the 1 axis (X). However, as outlined in the azimuth-elevation example, the angle in the horizontal plane is positive when measured from the 2 (Y) to the 1 (X) axis.

The coordinate rotation necessary to rotate the u, v, w system to the x, y, z system in Figure 3 are then,

$$\begin{pmatrix} \bar{x} - \bar{x}_p \\ \bar{y} - \bar{y}_p \\ c \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \bar{K}\,\underset{\sim}{P_1 R_3}(-\varkappa)\,\underset{\sim}{P_2 R_1}\left(\omega - \frac{\pi}{2}\right)\underset{\sim}{R}_3(-\psi) \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad , \tag{3.5}$$

where $\bar{K}$ is a projective constant relating object and image space. The results of the three matrix rotations can be expressed

$$\begin{pmatrix} \bar{x} - \bar{x}_p \\ \bar{y} - \bar{y}_p \\ c \end{pmatrix} = \bar{K} \begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad , \tag{3.6}$$

or

$$\begin{aligned} \bar{x} - \bar{x}_p &= \bar{K}\,(A_1 u + A_2 v + A_3 w)\,, \\ \bar{y} - \bar{y}_p &= \bar{K}\,(B_1 u + B_2 v + B_3 w)\,, \\ c &= \bar{K}\,(C_1 u + C_2 v + C_3 w)\,, \end{aligned} \tag{3.7}$$

and, dividing the first two equations by the expression for c we have the fundamental projective equations

$$\frac{\bar{x} - \bar{x}_p}{c} = \frac{A_1 u + A_2 v + A_3 w}{C_1 u + C_2 v + C_3 w} ,$$

$$\frac{\bar{y} - \bar{y}_p}{c} = \frac{B_1 u + B_2 v + B_3 w}{C_1 u + C_2 v + C_3 w} . \qquad (3.8)$$

Where, in the notation of Figure 3 [Brown, 1957, p. 7] ,

$$\begin{aligned}
A_1 &= -\cos\psi \cos\varkappa - \sin\psi \sin\omega \sin\varkappa , \\
A_2 &= \sin\psi \cos\varkappa - \cos\psi \sin\omega \sin\varkappa , \\
A_3 &= \cos\omega \sin\varkappa , \\
B_1 &= \cos\psi \sin\varkappa - \sin\psi \sin\omega \cos\varkappa , \\
B_2 &= -\sin\psi \sin\varkappa - \cos\psi \sin\omega \cos\varkappa , \\
B_3 &= \cos\omega \cos\varkappa , \\
C_1 &= \sin\psi \cos\omega , \\
C_2 &= \cos\psi \cos\omega , \\
C_3 &= \sin\omega .
\end{aligned} \qquad (3.9)$$

Therefore, if the stellar coordinates u, v, w can be computed for each stellar image, and the camera is fixed in astronomic azimuth elevation and roll, the camera orientation angles ψ, ω, and $\varkappa$, and the "precalibration" values for $\bar{x}_p$, $\bar{y}_p$ and c can be determined; the relationship between object and image space is defined.

If the orientation parameters ψ, ω, $\varkappa$, and principal point coordinates $\bar{x}_p$, $\bar{y}_p$, c are known or have been determined, the direction cosines u_u, v_u, w_u

to an unknown measured image may be found by transposing the orthogonal orientation matrix:

$$\begin{pmatrix} u_u \\ v_u \\ w_u \end{pmatrix} = \frac{1}{K} \begin{pmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{pmatrix} \begin{pmatrix} \bar{x}_u - \bar{x}_p \\ \bar{y}_u - \bar{y}_p \\ c \end{pmatrix} , \qquad (3.10)$$

where x_u and y_u are the measured coordinates of the unknown points.

The basic theory of camera reduction as outlined above is complicated by the fact that, in reality, the rays are not projected as straight lines due to lens distortion, the measured coordinates are not errorless, the principal distance and principal point coordinates are not known and the orientation angles ψ, ω, κ are not known. These effects and other parameters affecting the ideal projective equations are discussed in the following paragraphs. The final advanced plate reduction formulas are given in Section 3.22.

3.211 The Altitude-Azimuth System

The above system implies that we can physically determine the azimuth and elevation (hence the direction cosines u, v, w) to a stellar point for the epoch of observation. In this case, the epoch becomes quite significant since the star in its diurnal path or the satellite in its orbital path is continually changing. In Section 2 we outlined methods for determining the apparent position of star (α, δ). The first step is to convert the apparent α, δ of a stellar position corrected for diurnal aberration into direction cosines. The cartesian coordinate system is shown in Figure 5a. The direction cosines are

FIGURE 5a

THE RIGHT ASCENSION DECLINATION SYSTEM (X, Y, Z)

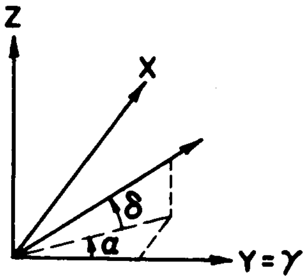


FIGURE 5b

THE RIGHT ASCENSION-DECLINATION AND HOUR ANGLE-DECLINATION SYSTEMS

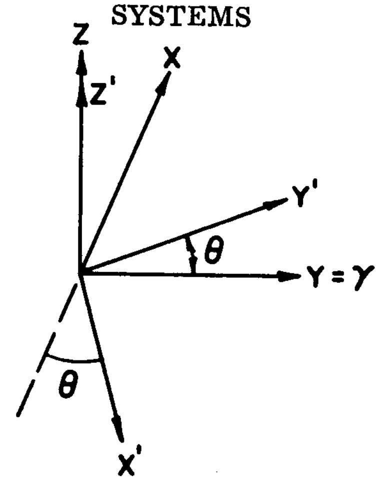
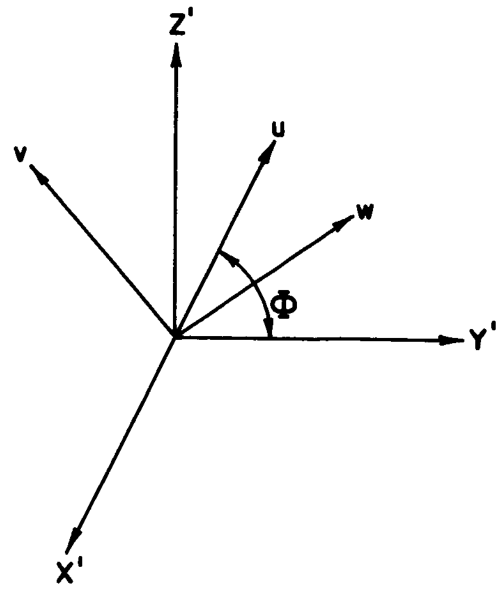


FIGURE 5c

THE HOUR ANGLE-DECLINATION AND AZIMUTH-ALTITUDE (u, v, w) SYSTEMS



$$
\begin{aligned}
X &= \cos\delta \ \sin\alpha \ , \\
Y &= \cos\delta \ \cos\alpha \ , \\
Z &= \sin\delta \ .
\end{aligned}
\tag{3.11}
$$

Following, the notation developed in Section 3.21 the X, Y, Z system is oriented at the observer's position with the Z axis defined by the true celestial pole, the Y axis in the direction of the true vernal equinox, and the X axis perpendicular to the Z and Y axes in the direction of increasing right ascension. Figure 5b shows the relationship between the X, Y, X, system and the X', Y', Z' or hour-angle, declination system. The X', Y', Z' system is topocentric with the Z' axis coincident with the Z axis, the Y' axis passes through the observer's astronomic meridian (free of polar motion), and the X' axis perpendicular to the Z' and Y' axis in the direction of positive hour-angle increase. Following the notation developed in Section 3.21 we can write

$$
\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \underset{\sim}{P}_1 \underset{\sim}{R}_3(-\theta) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad , \tag{3.12}
$$

where θ is the apparent local sidereal time. The implications inherent in determining θ accurately are discussed later in this section. Figure 5c depicts the relationship between the X', Y', Z' coordinate system and the u, v, w azimuth-altitude system which is described in Section 3.21. Mathematically, the relationships are

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \underset{\sim}{P_1}\,\underset{\sim}{P_2}\,\underset{\sim}{R_1}\,(\Phi - \frac{\pi}{2}) \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} , \qquad (3.13)$$

where Φ is the astronomic latitude of the observer. The above expressions can be written [Brown, 1957, p. 7],

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\sin\Phi & \cos\Phi \\ 0 & \cos\Phi & \sin\Phi \end{pmatrix} \begin{pmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sin\alpha \ \cos\delta \\ \cos\alpha \ \cos\delta \\ \sin\alpha \end{pmatrix}, \qquad (3.14)$$

where the u', v', w' system refers to the unrefracted direction of a plate image in the altitude azimuth system. Mathematically,

$$\begin{aligned} u' &= \sin\overline{\psi} \ \cos\overline{\omega} , \\ v' &= \cos\overline{\psi} \ \cos\overline{\omega} , \\ w' &= \sin\overline{\omega} , \end{aligned} \qquad (3.15)$$

where $\overline{\psi}$, $\overline{\omega}$ are the azimuth and altitude of a specific image. The zenith distance ($\overline{Z}_R$) of the refracted ray can then be expressed as the unrefracted zenith distance ($90 - \overline{\omega}$ or Z) minus the astronomical refraction (ΔZ) as developed in Section 2. Therefore, the direction cosines to the observed ray can be written

$$\begin{aligned} u &= \sin\overline{\psi} \ \sin(Z - \Delta Z) , \\ v &= \cos\overline{\psi} \ \sin(Z - \Delta Z) , \\ w &= \cos(Z - \Delta Z) . \end{aligned} \qquad (3.16)$$

As Brown has shown we can express the above relationship as [Brown, 1964, p.7]

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \cos \Delta Z & 0 & -\sin\psi \sin \Delta Z \\ 0 & \cos \Delta Z & -\cos\psi \sin \Delta Z \\ \sin\overline{\psi} \sin \Delta Z & \cos\overline{\psi} \sin \Delta Z & \cos \Delta Z \end{pmatrix} \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} , \quad (3.17)$$

where $\overline{\psi}$ may be found from

$$\overline{\psi} = \arctan \left(\frac{u'}{v'}\right) \quad . \quad (3.18)$$

Therefore, at least theoretically, we have related the observed positions of the stars and the azimuth-altitude system of the camera. A problem still exists, however, in determining the sidereal time and astronomic latitude.

The sidereal time used in the development outlined above represents the angle between the true vernal equinox, and the observer's meridian. The astronomic latitude is the angle between the mean terrestrial equator (free of polar motion) and the true normal (vertical) of the observer (positive in the northern hemisphere). Unfortunately, the precise values of sidereal time and astronomic latitude are not immediately available and provisional values may be used in plate reduction. The values used in the plate reduction will define a provisional azimuth-altitude system. Thus, if star coordinates in the right ascension-declination system are used in the plate reduction, altitude and azimuth will be defined by the provisional values of sidereal time and latitude used in equation 3.14. However, if the same provisional values of sidereal time and astronomic latitude are used to determine the star's right ascension and declination from the altitude-azimuth coordinates, the original right ascension and declination will be obtained .

Since the provisional values of sidereal time and astronomic latitude used in plate reduction will depend on the procedures adopted by a specific agency, the author recommends that agencies using the advanced plate reduction procedure to provide satellite observation data to the Geodetic Satellite Data Center for future geodetic investigation refer the satellite position to the right ascension-declination system. If the satellite's position is given in the azimuth-altitude system, the following information must also be provided to allow further investigation:

1. The type of projective equations used and the orientation of the coordinate systems. For example, the basic definitions outlined above are only one method of orientation. Another agency may use an entirely different system.

2. The type of time (U. T.) used should be specified exactly and the type of time and the method used in determining the sidereal time must be defined.

3. The latitude used in determining the u, v, w coordinates must be specified. For example, the geodetic latitude of the Minitrack Optical Tracking Station at College, Alaska will vary as shown below for various geodetic datums:

φ	DATUM
$64^\circ 52' 19''.721$	North American (1927)
$64^\circ 52' 18''.610$	Mercury
$64^\circ 52' 18''.530$	Vangard

3.212 Plate Measurement and Lens Distortion Corrections

Now that we have examined the elements affecting the object space, we must at least briefly outline the factors affecting the measured coordinates on the photographic plate. The study of the techniques of plate measurement and lens distortion corrections for ballistic cameras have been outlined in exhaustive studies by Schmid and Brown. As "users" of the data, we are interested in the corrections only in the sense that the corrections are applied. Consequently, the content of the following paragraphs are limited to what the corrections are and a method (from Brown and Harp) in which they may be applied [Brown, 1957, pp. 79-82; Harp, 1966, pp. 13-17].

The objective of plate measurement is to ascertain the values of $\bar{x}$, $\bar{y}$ of a number of known and unknown stellar images as accurately as possible. At present, the instrument most commonly used to measure the $\bar{x}$, $\bar{y}$ coordinates is the Mann Comparator. The Mann Comparator when calibrated using a carefully calibrated grid and using the arithmetic mean of two sets of four measurements each is capable of measuring the coordinates of an individual image with an accuracy of ± 1.5 microns [Schmid, 1966, pp. 18-19]. The corrections for comparator errors are,

1. Image measuring bias. The plate images are not perfectly defined dots, but are slightly distorted figures which are generally elongated (in a fixed camera exposure) along the diurnal stellar path and along the satellite's orbital path. Therefore, during plate measurement, the position of the center of the image may be biased. Most agencies minimize the bias effect

by measuring each image in the "direct" position, rotate the plate 180°, and measure the image in the "reverse" position, and then average the plate measurements for use in plate reduction.

2. Nonperpendicularity of the comparator axis. The corrections (dx_1 and dy_1) are given by

$$
\begin{aligned}
dx_1 &= \bar{y}' \sin e \,, \\
dy_1 &= \bar{y}' \,(\cos e - 1) \,,
\end{aligned}
\tag{3.19}
$$

where $\bar{y}'$ is the average $\bar{y}$ measurement of an image, and e is the non-perpendicularity of the $\bar{y}$ axis with respect to a true perpendicular to the $\bar{x}$ axis.

3. Weave of the guides of the comparator axis. The comparator axes may "weave" in a manner that can be detected by the comparator calibration procedures. If the amount of weave is known the comparator measurement corrections may be expressed as

$$
\begin{aligned}
dx_2 &= f_1 \,(\bar{y}') \,, \\
dy_2 &= f_2 \,(\bar{x}') \,,
\end{aligned}
\tag{3.20}
$$

where the functions are determined by comparator calibrations and are a function of the dial reading of the comparator.

4. Periodic screw error. The comparator axes may introduce periodic variations in a coordinate measurement depending on the position of the screw from a geometrical plate center $(\bar{x}^\circ, \bar{y}^\circ)$. The correction is

$$
\begin{aligned}
dx_3 &= a_1 \sin 2\pi \,(\bar{x}' - \bar{x}^\circ) \,, \\
dy_3 &= b_1 \sin 2\pi \,(\bar{y}' - \bar{y}^\circ) \,,
\end{aligned}
\tag{3.21}
$$

where a_1 and b_1 are constants determined during comparator calibration.

5. Secular screw error. The comparator axes may introduce a secular error which may be determined from

$$\begin{aligned} dx_4 &= g_1\ (\bar{x}')\ , \\ dy_4 &= g_2\ (\bar{y}')\ , \end{aligned} \tag{3.22}$$

where the functions are determined by comparator calibration and are a function of the comparator dial reading.

Finally, the corrected $\bar{x}''\ \bar{y}''$ measurements of the measured image are,

$$\begin{aligned} \bar{x}'' &= \bar{x}' + dx_1 + dx_2 + dx_3 + dx_4 \quad , \\ \bar{y}'' &= \bar{y}' + dy_1 + dy_2 + dy_3 + dy_4 \quad . \end{aligned} \tag{3.33}$$

The above procedure is then repeated for each point.

The measured coordinates, however, are still affected by lens distortions. The symmetric radial lens distortion can be expressed as [American Society of Photogrammetry, 1966, p. 190],

$$\begin{aligned} \delta x &= \bar{x}''(K_1\ r^2 + K_2\ r^4 + K_3\ r^6 + \ldots\ ) \quad , \\ \delta y &= \bar{y}''(K_1\ r^2 + K_2\ r^4 + K_3\ r^6 + \ldots\ ) \quad , \end{aligned} \tag{3.24}$$

where r is the radial distance from the principal point to the measured point and, K_1, K_2, ... are coefficients of the lens or camera system used. Generally three terms of the expansion are sufficient for practical applications. Further discussion of how these coefficients may be determined will be discussed later.

Finally, we must consider the displacement of the image due to tangential distortion. The correction Δx, Δy for tangential distortion is [Brown, 1964, p. 58],

$$\Delta x = -P' \sin \hat{\varphi} \quad , \qquad\qquad (3.25)$$
$$\Delta y = P' \cos \hat{\varphi} \quad ,$$

where $\hat{\varphi}$ is an angle measured clockwise from the positive axis to the axis of maximum tangential distortion, and P' is given by

$$P' = J_1 r^2 + J_2 r^4 + \ldots \quad , \qquad\qquad (3.26)$$

and J_1, J_2 ... (and $\hat{\varphi}$) are tangential distortion coefficients for a given camera that must be determined by camera calibration. Generally, two "J" terms are sufficient for practical application.

If the constants of distortion outlined above are known for a given camera, the measured coordinates $\bar{x}'' \bar{y}''$ may be corrected by

$$\bar{\bar{x}} = \bar{x}'' + \delta x + \Delta x \quad , \qquad\qquad (3.27)$$
$$\bar{\bar{y}} = \bar{y}'' + \delta y + \Delta y \quad .$$

In the above expressions for radial and tangential distortion, we assume the principal point of the camera is known. Of course, this is not the case since $\bar{x}_p$ and $\bar{y}_p$ are not known precisely until after plate calibration. To overcome this difficulty we may either use a nominal plate center obtained from a previous adjustment and allow the precalibrated values of K_1, K_2, K_3, J_1, J_2, $\hat{\varphi}$ to be enforced or calibrate the camera during each plate reduction by successively reiterating the above expressions in the general equations to obtain a least squares solution.

3.22 The Final Projective Equations

The basic projective equations at the first of this section can now be written as

$$\frac{\bar{x}'' + \delta x + \Delta x - \bar{x}_p}{c} = \frac{A_1 u + A_2 v + A_3 w}{C_1 u + C_2 v + C_3 w} ,$$

$$\frac{\bar{y}'' + \delta y + \Delta y - \bar{y}_p}{c} = \frac{B_1 u + B_2 v + B_3 w}{C_1 u + C_2 v + C_3 w} . \tag{3.28}$$

In the above equations, the left-handed side of the equations contain the star image and camera parameters and the right-hand side contains the camera orientation stellar parameters.

Before considering how the equations may be solved, we must consider the parameters in the equations and how the parameters may be determined. As outlined earlier in Sections 2 and 3 the known parameters are ,

1. The measured values of the image on the photographic plate ($\bar{x}''$, $\bar{y}''$) corrected for comparator errors as outlined in Section 3.21.

2. The right ascension and declination (α, δ) of a star taken from a star catalogue and updated to the apparent place including diurnal aberration as outlined in Section 2.

3. The sidereal time (θ) obtained from a timing record associated with the plate exposure.

4. The latitude of the station (φ) which is assumed constant for a station.

The unknown or approximately known parameters explicit in the equations are

1. The elements of exterior orientation (ψ, ω, κ) of the fixed camera system.

2. The elements of inner orientation ($\bar{x}_p$, $\bar{y}_p$, c) of the camera system.

3. The coefficients of distortion J_1, J_2, $\hat{\varphi}$, K_1, K_2 and K_3 outlined in Section 3.212.

4. The coefficients of refraction (η_1, η_2, η_3, η_4) as outlined in Section 2. The author indicated for illustrative purposes that the refraction coefficients could be determined explicitly; however, it is possible to consider the coefficients as constrained parameters for purposes of plate reduction.

Thus, for the two projective equations available for each point we have (in this example) sixteen unknown parameters. The parameters, however, are not really unknown since we have attached physical significance to each parameter. For example, the coefficients of refraction determined from techniques outlined in Section 2 are probably accurate to one or two percent [Brown, 1964, p. 21]. The coefficients of distortion and inner orientation elements may be available from a previous calibration (for a given camera) and be considered to be constrained or known parameters.

Other unknown or constrained parameters may also be introduced. For example, the star coordinates (α, δ) may be considered to be constrained by the standard errors given in the catalogue used. In fact, if a star image exists on the photograph, but catalogued coordinates are not available for the star's position, the star's coordinates (now unknown parameters) may be considered to have zero weight but carried in the adjustment of the projective equations.

Now that we have outlined the final projective equations we must examine, at least generally, methods for solving the equations.

3.23 Solution of the Projective Equations

As mentioned earlier (Section 3.21), the solution of the projective equations can be regarded as a two-step process. First, the unknown or constrained parameters must be determined by using known stellar points, and second, the position to an unknown image (i.e., a satellite image) must be determined, using the parameters determined by the first step.

At the present, basically two philosophies are used for the first step of the process (i.e., determining the unknowns or constrained parameters). The first technique is to completely calibrate the camera system each time a plate is reduced, and the second technique is to completely calibrate one plate from a given camera and then use certain coefficients (for example, K_1, K_2, K_3, J_1, J_2, etc.) determined from the complete calibration as constants in further plate reductions. The technique used is determined by the agency performing the plate reduction. The first technique involves quite lengthy computational procedures compared to the second technique but is certainly the most rigorous. Obviously, the second technique assumes the "camera constants" determined from the plate calibration do not change significantly with time or different meteorological conditions at the camera site. The author was unable to find any published literature indicating the assumption is valid.

The next problem is the actual least squares solution of the projective equations. The precise formulas used in the least squares adjustment will, of course, differ depending on what parameters are assumed "known" and the parameters that are unknown or constrained. The author's intent is simply to give an outline of the basic technique since the basic technique and ramification of using different matrix partitioning technique are given in exhaustive studies by Brown and Schmid [Brown, 1957; Schmid, 1959].

The basic projective equations can be expressed as

$$\bar{e}_x = \frac{\bar{x}'' + \delta x + \Delta x - \bar{x}_p}{c} - \frac{A_1 u + A_2 v + A_3 w}{C_1 u + C_2 v + C_3 w} \quad ,$$

$$\bar{e}_y = \frac{\bar{y}'' + \delta y + \Delta y - \bar{y}_p}{c} - \frac{B_1 u + B_2 v + B_3 w}{C_1 u + C_2 v + C_3 w} \quad , \qquad (3.29)$$

where $\bar{e}_x$ and $\bar{e}_y$ indicate the amount of "error" in the equations. The functional equations are

$$f_1 \ (\bar{x}'', \bar{y}'', \alpha, \delta, \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_n) = 0 \quad ,$$

$$f_2 \ (\bar{x}'', \bar{y}'', \alpha, \delta, \bar{u}_1, \bar{u}_2, \ldots, u_n) = 0 \quad , \qquad (3.30)$$

where $\bar{u}_i$ indicates the adjustable parameter (ψ, ω, κ, $\bar{x}_p$, ..., etc.) in the basic projective equations.

If, for simplicity, we assume the only parameters to be adjusted are ψ°, ω°, κ°, $\bar{x}_p^\circ$, $\bar{y}_p^\circ$ and $c_\circ$ where the zero superscript denotes an approximate value of the parameter, we can linearize the projective equations by performing a Taylor's expansion about the approximate values.

The linearization will give

$$v_x + b_1 \delta\psi + b_2 \delta\omega + b_3 \delta\kappa + b_4 \delta\bar{x}_p + b_5 \delta\bar{y}_p + b_6 \delta c + \bar{E}_x = 0 \quad ,$$

$$v_y + b'_1 \delta\psi + b'_2 \delta\omega + b'_3 \delta\kappa + b'_4 \delta\bar{x}_p + b'_5 \delta\bar{y}_p + b'_6 \delta c + \bar{E}_y = 0 \quad . \qquad (3.31)$$

The omission of second order terms is significant since the approximate parameters may differ significantly from the adjusted values. However, as will be shown, the solution will be reiterated to correct the approximate parameters until the differences (in the sense adjusted parameters minus approximate parameters) are differentially small. The delta (δ) quantities for the adjustable parameters are given in the sense

$$\psi_a = \psi^\circ + \delta\psi \quad , \tag{3.32}$$

where ψ_a indicates the adjusted value of ψ°. The v_x and v_y terms are the corrections to the original measured plate coordinates ($\bar{x}''$, $\bar{y}''$) as shown below:

$$\begin{aligned} \bar{x}_a'' &= \bar{x}'' + v_x \quad , \\ \bar{y}_a'' &= \bar{y}'' + v_y \quad , \end{aligned} \tag{3.33}$$

where $\bar{x}_a''$, $\bar{y}_a''$ indicate the adjusted values of $\bar{x}''$ and $\bar{y}''$. The b_1, b_2, ..., b_5', b_6' terms are the partial derivatives of the expanded projective equation with respect to the parameter to be adjusted, and $\bar{E}_x$ and $\bar{E}_y$ denote the numerical values of $\bar{e}_x$ and $\bar{e}_y$ in equation 3.29 evaluated with approximate parameters (ψ°, ω°, etc.).

To form the b_i and b_i' terms we must first form an auxiliary relationship using the approximate values of the parameters for each image [Brown, 1964, pp. 24-41; Harp, 1966, pp. 49-51]:

$$\begin{pmatrix} m^\circ \\ n^\circ \\ q^\circ \end{pmatrix} = \begin{pmatrix} A_1{}^\circ & A_2{}^\circ & A_3{}^\circ \\ B_1{}^\circ & B_2{}^\circ & B_3{}^\circ \\ C_1{}^\circ & C_2{}^\circ & C_3{}^\circ \end{pmatrix} \begin{pmatrix} u^\circ \\ v^\circ \\ w^\circ \end{pmatrix} \quad . \tag{3.34}$$

The zero superscripts indicate the terms evaluated with the approximations. Then let

$$
\begin{aligned}
Q^\circ &= \frac{c^\circ}{q^\circ}, \quad N = \frac{n^\circ}{q^\circ}, \quad M^\circ = \frac{m^\circ}{q^\circ} \quad , \\
S &= -(A_2^\circ - C_2 M^\circ)u^\circ + (A_1^\circ - C_1^\circ M^\circ)v^\circ \quad , \\
h &= u^\circ \sin\psi^\circ + v^\circ \cos\psi^\circ \quad , \\
S' &= (A_3^\circ - C_3^\circ M^\circ)h + [M^\circ \cos v^\circ + C_3{}^\circ \sin \varkappa^\circ]w^\circ \quad , \\
S_1 &= -(B_2{}^\circ - C_2{}^\circ N^\circ)u^\circ + (B_1{}^\circ - C_1{}^\circ N^\circ)v^\circ \quad , \\
S_1' &= (B_3{}^\circ - C_3{}^\circ N^\circ)h + [N^\circ \cos\omega^\circ + C_3{}^\circ \cos\varkappa^\circ]w^\circ \quad .
\end{aligned}
\tag{3.35}
$$

The the coefficients $b_1, \ldots, b_6'$ become

$$
\begin{aligned}
b_1 &= SQ^\circ & b_1' &= S_1 Q^\circ \\
b_2 &= S'Q^\circ & b_2' &= S_1' Q^\circ \\
b_3 &= -c^\circ N^\circ & b_3' &= c^\circ M^\circ \\
b_4 &= -1 & b_4' &= 0 \\
b_5 &= 0 & b' &= -1 \\
b_6 &= -M^\circ & b_6' &= -N^\circ
\end{aligned}
\tag{3.36}
$$

Now, the linearized projective equations (3.31) for one star image can be written in matrix notation as

$$
\underset{\sim}{V}_i + \underset{\sim}{B}_i \, \underset{\sim}{\Delta} = \underset{\sim}{E}_i \quad , \tag{3.37}
$$

where

$$\left.\begin{aligned} \underset{\sim}{V}_i &= \begin{pmatrix} v_x \\ v_y \end{pmatrix} \\ \underset{\sim}{B}_i &= \begin{pmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ b_1' & b_2' & b_3' & b_4' & b_5' & b_6' \end{pmatrix} \\ \underset{\sim}{E}_i &= \begin{pmatrix} \overline{E}_x \\ \overline{E}_y \end{pmatrix} \end{aligned}\right\} , \tag{3.38}$$

$$\underset{\sim}{\Delta} = \begin{pmatrix} \delta\psi \\ \delta\omega \\ \delta\kappa \\ \vdots \\ \delta c \end{pmatrix} , \tag{3.39}$$

and the subscript (i) indicates the matrix formula is for the i^{th} star image. If a star trace consisting of n star images was measured for one star (j), we would have

$$\overline{\underset{\sim}{V}}_j = \begin{pmatrix} \underset{\sim}{V}_1 \\ \underset{\sim}{V}_2 \\ \vdots \\ \underset{\sim}{V}_n \end{pmatrix} \quad \overline{\underset{\sim}{B}}_j = \begin{pmatrix} \underset{\sim}{B}_1 \\ \underset{\sim}{B}_2 \\ \vdots \\ \underset{\sim}{B}_n \end{pmatrix} \quad \overline{\underset{\sim}{E}}_j = \begin{pmatrix} \underset{\sim}{E}_1 \\ \underset{\sim}{E}_2 \\ \vdots \\ \underset{\sim}{E}_n \end{pmatrix} . \tag{3.40}$$

Thus for one trace of the jth star we can write

$$\overline{\underset{\sim}{V}}_j + \overline{\underset{\sim}{B}}_j \underset{\sim}{\Delta} = \overline{\underset{\sim}{E}}_j . \tag{3.41}$$

At this point it must be mentioned that if the star coordinates are assumed to be "weighted" the correction for the star coordinates would be

$$\dot{\Delta} = \begin{pmatrix} \delta\alpha_j \\ \delta\delta_j \end{pmatrix} \quad , \tag{3.42}$$

where $\dot{\Delta}$ is the correction vector to the approximate right ascension and declination used in the adjustment. The if $\dot{B}_j$ is the linearized expansion of the projective equations with respect to right ascension and declination, equation 3.41 would become

$$\underset{\sim}{\overline{V}_j} + \underset{\sim}{\overline{B}_j}\, \underset{\sim}{\Delta} + \underset{\sim}{\dot{B}_j}\, \underset{\sim}{\dot{\Delta}} = \underset{\sim}{\overline{E}_j} \quad . \tag{3.43}$$

However, for our simplified case, we assume α, δ to be known quantities so equation 3.41 will be used. We could extend the analogy used in obtaining equation 3.43 to all m star traces, where the final $\underset{\sim}{\overline{\overline{V}}}$ matrix for the plate would consist of a column matrix consisting of $\underset{\sim}{\overline{V}_1}$, $\underset{\sim}{\overline{V}_2}$, $\overline{V}_m$. However, instead of manipulating the unwieldy final matrices for the entire plate, the individual matrices for each star image may be used to form the normal equations and determined the solution [Harp, 1966, p. 51]. The equations are as follows for n images per star trace and m stars. If we assume that n images of one star are available each image will have an equation (from 3.37)

$$\underset{\sim}{V_i} + \underset{\sim}{B_i}\, \underset{\sim}{\Delta} = \underset{\sim}{E_i} \quad , \tag{3.44}$$

then let

$$\underset{\sim}{N_j} = \sum_{i=1}^{n} \underset{\sim}{B_i^T}\, \underset{\sim}{W_i}\, \underset{\sim}{B_i} \quad , \tag{3.45}$$

and

$$\underset{\sim}{C_j} = \sum_{i=1}^{n} \underset{\sim}{B_i^T}\, \underset{\sim}{W_i}\, \underset{\sim}{E_i} \quad , \tag{3.46}$$

where $\underset{\sim}{W}_i$ is the weight matrix of the i^{th} stellar image. Then for m stars

$$
\begin{aligned}
\underset{\sim}{\overline{N}} &= \sum_{j=1}^{m} \underset{\sim}{N}_j \quad , \\
\underset{\sim}{\overline{C}} &= \sum_{j=1}^{m} \underset{\sim}{C}_j \quad .
\end{aligned}
\tag{3.47}
$$

The solution is

$$
\underset{\sim}{\Delta} = (\underset{\sim}{\overline{N}} + \underset{\sim}{\dot{W}})^{-1} \; (\underset{\sim}{\overline{C}} - \underset{\sim}{\dot{W}} \; \underset{\sim}{\dot{E}}) \quad , \tag{3.48}
$$

where $\underset{\sim}{\dot{W}}$ is the weight matrix of the adjustable parameters (ψ, ω, $\varkappa$, etc.). The discrepancy vector $\underset{\sim}{\dot{E}}$ is defined as

$$
\underset{\sim}{\dot{E}} = \begin{pmatrix} \underline{\psi} & -\psi^{\circ} \\ \underline{\omega} & -\omega^{\circ} \\ \underline{\varkappa} & -\varkappa^{\circ} \\ \vdots & \\ \underline{c} & c^{\circ} \end{pmatrix} \quad . \tag{3.49}
$$

The zero superscripts denote an a priori or measured value and $\underline{\psi}$ denotes an arbitrary approximation for the iteration (for the initial iteration the terms in $\underset{\sim}{\dot{E}}$ will be equal to zero). Adjustment to approximate values can then be made dependent on values determined from $\underset{\sim}{\Delta}$ matrix. For example, if the k^{th} iteration yielded values of ψ_k , ω_k , . . . , c_k , then

$$
\begin{aligned}
\psi_{k+1} &= \psi_k + \delta\psi \; , \\
\omega_{k+1} &= \omega_k + \delta\psi \; , \\
&\vdots \\
c_{k+1} &= c_k + \delta c \; .
\end{aligned}
\tag{3.50}
$$

In order to minimize errors which may be introduced by neglecting higher order terms in the Taylor's expansion of the projective equation, an iterative process is employed until the values of $\underset{\sim}{\Delta}$ are small. After each iteration the approximate values of the parameters will change and hence the $\underset{\sim}{\dot{E}}$ vector will not be zero. When $\underset{\sim}{\Delta}$ is sufficiently small, the values of ψ, ω, . . . , c used in the last iteration will represent the adjusted values of the adjustable parameters and the residuals of the measured values $\underset{\sim}{V}_i$ of the i^{th} star will be equal to $\underset{\sim}{E}_i$.

After the adjustment process the direction to an unknown point u_u, v_u, w_u will be given by the inversion of the basic projective equations given by equation 3.10 where the adjusted values of the parameters will be used and the measured plate coordinates will be corrected for comparator errors and lens distortion.

Finally, we may ascertain the covariance matrix for the adjusted parameters from

$$\Sigma = (\underset{\sim}{\overline{N}} + \underset{\sim}{\dot{W}})^{-1} \quad . \tag{3.51}$$

Various agencies use different procedures in determining the standard deviation of "output" directions (for example right ascension and declination), therefore, the author will discuss the specific procedures in Section 4.

One point which may not be clear from the above simplified example, is that uncatalogued stars may be carried through the advanced plate reduction by treating the unknown stellar coordinates (see equation 3.43) as observations of zero weight [Brown, 1964, pp. 45-46].

3.24 Simplification of the General Case

The general projective equations as the author has emphasized earlier, are assumed to have physical significance. However, long before the projective equations were developed, astronomers used a "simple" system to determined stellar positions and changes in stellar position (proper motions). The simplified procedure is called the "Turner's Method," and was developed by H. H. Turner in 1893 [Turner, 1893].

Development of the Turner method can be accomplished by re-examination of the basic projective equations (equation 3.6). Figure 6 illustrates the basic topocentric right ascension-declination system with a plane tangent to the celestial sphere at α_o, δ_o. The tangent plane describes a coordinate system ξ, η, and is called the "standard coordinate" system. As shown in Figure 6, the η axis lies in the plane defined by the Z axis and the meridian defined by α_o. The ξ axis lies in a plane which is perpendicular to the η axis.

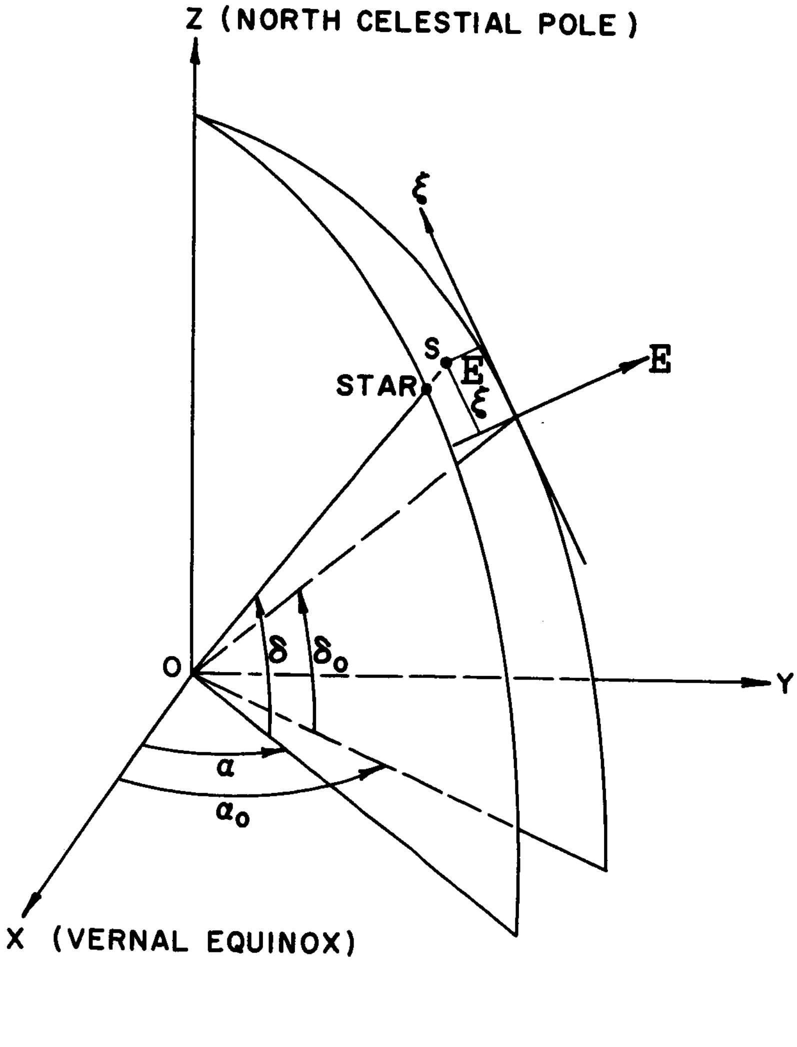
The ξ, η coordinates of an arbitrary stellar image (S) must be determined by projecting the right ascension-declination coordinates (α, δ) of a star image on the tangent plane. The one agency (The Smithsonian Astrophysical Observatory) currently using the simplified procedure uses the following projection for determining the standard coordinates ξ, η of an arbitrary star image [Moss et al., 1966, p. 24],

$$\xi = \frac{\cot\delta \sin(\alpha - \alpha_o)}{\sin\delta_o + \cos\delta_o \cot\delta \cos(\alpha - \alpha_o)} ,$$

$$\eta = \frac{\cos\delta_o - \cot\delta \sin\delta_o \cos(\alpha - \alpha_o)}{\sin\delta_o + \cot\delta \cos\delta_o \cos(\alpha - \alpha_o)} . \tag{3.52}$$

FIGURE 6

THE CELESTIAL SPHERE AND TANGENT PLANE



Mueller indicates the projection is an approximation valid only for a limited area around the (α_o, δ_o) origin [Mueller, 1964, p. 311].

The coordinates of a star image on the tangent plane are ξ, η, 1 since the third axis of the plane is directed toward the projection center. The tangent plane coordinates could be related to a camera plate coordinate system referred to the projection center ($\bar{x}$, $\bar{y}$, c) in the same general method outlined in section 3.21. In this case, however, the projective equations (analogous to equation 3.8) become

$$\begin{aligned} \bar{x} - \bar{x}_p &= \frac{a_1\xi + b_1\eta + c_1}{d\xi + e\eta + f}\,(c) \quad , \\ \bar{y} - \bar{y}_p &= \frac{a_2\xi + b_2\eta + c_2}{d\xi + e\eta + f}\,(c) \quad , \end{aligned} \tag{3.53}$$

where a_i, b_i, c_i, d, e, and f are "plate constants" which result from the orientation matrices necessary to relate the camera plate and tangent plane coordinate systems.

If we further assume that the tangent plane and camera plate are parallel, the above equations become [Turner, 1893]

$$\begin{aligned} \bar{x} - \bar{x}_p &= a_1\,\xi + b_1\eta + c_1 \quad , \\ \bar{y} - \bar{y}_p &= a_2\,\xi + b_2\eta + c_2 \quad . \end{aligned} \tag{3.54}$$

The a_i, b_i, and c_i coefficients in the above equations are called the "linear plate constants" and the equations assume that a straight line on the tangent plane is projected onto the $\bar{x}$, $\bar{y}$ plane as a straight line.

The solution of the linear plate constant method is very straightforward. Each point generates two equations and each plate has six unknowns (a_1, b_1, c_1, a_2, b_2, c_2). Since, in the simplified case, the plate constants are not "observed" parameters, we have f degrees of freedom:

$$f = 2n - 6 , \tag{3.55}$$

where n is the number of stellar images. From the above equation, it is obvious that three stars will give a minimum or unique solution and four or more stars will allow a least squares solution to be accomplished.

If the "linear plate constant" method (i.e., equation 3.54) is used for plate reduction, the following factors must be considered:

1. The projection equation used to find the standard coordinates is an approximation valid only for a limited area around the (α_o, δ_o) origin.

2. The equations are based on the assumption that the tangent plane (ξ, η) is parallel to the camera plate ($\bar{x}$, $\bar{y}$).

3. The measured coordinates $\bar{x}$, $\bar{y}$ are assumed to be unaffected by lens distortions. The distortions as shown in section 3.212 are dependent on the radial distance of the measured image from the theoretical plate center.

Thus, the assumptions will be valid if very narrow fields of view are used and the lens distortions are small. However, Brown has indicated that, in general, this assumption is not true for ballistic cameras [Brown, 1964, p. 89]. Eichhorn has outlined procedures to theoretically determine the systematic error which can be expected due to the inaccuracy of this method (and more elaborate methods) [Eichhorn, 1963].

The practical solution of the plate constant method is essentially a six-step process:

1. First,the origin (α_o, δ_o) of the reference plane (ξ, η) must be determined. The approximate values may be obtained by using the geometric center of the plate (if fiducial marks are available) and the stellar position α_o, δ_o of the geometrical center can then be estimated if the α, δ of stars near the geometric center can be identified.

2. Second, the standard coordinates of the known stars selected must be computed from the projection used.

3. The known star images ($\bar{x}_i$, $\bar{y}_i$), the plate center ($\bar{x}_p$, $\bar{y}_p$), and unknown satellite images ($\bar{x}_s$, $\bar{y}_s$) must be measured. As in the general case, this must be done as precisely as possible.

4. The observation equations of the form

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} a & b \\ a' & b' \end{pmatrix} \begin{pmatrix} \bar{x}-\bar{x}_p \\ \bar{y}-\bar{y}_p \end{pmatrix} + \begin{pmatrix} c' \\ c'' \end{pmatrix} , \qquad (3.56)$$

must be established for the number of observed stars available. The equations can then be solved and the linear plate constants determined by a simple least squares solution of the observation equations. A numerical example of the solution has been published; consequently, the solution will not be outlined here [Mueller, 1964, pp. 313-314].

5. After the plate constants have been determined from the least square solution, the standard coordinates (ξ_s, η_s) of the unknown satellite images ($\bar{x}_s$, $\bar{y}_s$) can be determined from equation 3.56.

6. Finally the direction to the unknown satellite images (α_s, δ_s) can be found from equation 3.52,

$$\tan(\alpha_s - \alpha_o) = \frac{\xi_s}{\cos\delta_o - \eta_s \sin\delta_o} ,$$

$$\tan\delta_s = \frac{(\sin\delta_o + \eta_s \cos\delta_o)\cos(\alpha_s - \alpha_o)}{\cos\delta_o - \eta_s \sin\delta_o} . \tag{3.57}$$

The author was unable to find statistical analyses of the accuracy of the plate constant method using moderately short focal length (ballistic) cameras. The method, compared to the general solution, is very simple and avoids elaborate computational techniques. However, the method is limited since the equations used assume that the tangent plane is parallel to the photograph plane and only narrow fields of view should be used.

3.3 TIMING PROCEDURES

Precise timing and, more specifically, precise time coordination, provide the very basis of determining a "simultaneous" satellite observation. Markowitz has indicated that satellite observing stations can currently synchronize a clock to an accuracy of about ± one millisecond by use of coordinated transmissions from the United States, the United Kingdom, Argentina, Australia, Canada, Japan, South Africa, and Switzerland [Markowitz, 1964c]. An accuracy of one millisecond is required in observation of satellites for geodetic purposes [Markowitz, 1963, p. 217].

If we assume the U. T. C. epoch of the coordinate transmitting station to be "synchronized" to within 1 ms of other transmitting stations, we still must consider techniques necessary to determine the satellite observation epochs at the observing station. Presently, two basic techniques are used.

The first technique is to use a radio receiver to receive standard time broadcasts (i. e., such as WWV) at the station. The received time signals are used to initially set the epoch of the station clock by recording and comparing the radio transmission and station clock signals. Monitoring the time signal broadcasts, the epoch and rate of the station clock may be determined and then the station clock may be used to correlate the epoch of shutter action by recording and comparing the station signals and signals produced by the shutter or by recording the epoch of observation of the film plate.

The second technique employs a travelling clock which is taken to the observing station to initially set the station clock. After the epoch of the station clock has been determined, VLF radio signals may be monitored to control day-to-day variations. The station clock may then be used to determine shutter action times as in the first technique.

The two basic techniques outlined above are listed in order of increasing accuracy. The first method requires propagation delay corrections to be applied to the received radio signals. Propagation characteristics of electromagnetic waves is, of course, an extremely complex study. Markowitz has indicated that high frequency radio signals may be used for timc synchronization to ±1 ms at distances up to 1000 km [Markowitz, 1964a]. Very low frequencies, on the other hand, may be used for synchronization with a probable error of time of arrival of a single pulse of 90 microseconds and an uncertainty in propagation time of 0.5 ms by using techniques developed at the U. S. Naval Observatory [Markowitz, 1964a]. In general, high frequency time signal broadcasts are preferred in determining precise time comparisons at a station, and VLF broadcasts are preferred for monitoring the frequency of the station clock [Preuss, 1966, p. 126]. Very low frequencies provide frequency measurement standards for accurate monitoring of day-to-day variations and are capable of providing accuracies to 1 part in 10^{11} for a twenty-four hour period [Markowitz, 1964a]. The second "travelling clock" technique provides the method of highest precision. The precision is estimated to be much less than 1 millisecond with respect to the time standard (for example, the transmitted signals of WWV) [Satellite Triangulation in the Coast and Geodetic Survey, 1965, pp. 6-7].

The above paragraphs briefly review the generalities pertaining to timing procedures; however, it is now necessary to relate the timing procedures to the specific type of satellite observed and the mode of operation of the camera system.

3.31 Active Satellite Timing Procedures

The satellite flash times for the GEOS-A satellite were monitored and controlled by the Applied Physics Laboratory (APL) of the John Hopkins University. The APL published "GEOS-A Clock Calibration" bulletins which list the satellite clock offset relative to WWV (U. T. C.) [APL, 1966]. The satellite was then triggered to flash on a satellite clock minute marker and then either four or six more times at four second intervals. Since the flash was triggered on the minute (or second) mark the flash will occur subsequent to the trigger as shown in Figure 7. Satellite clock rates were then published in the APL bulletins to correct the flash time (governed by the satellite clock) to WWV (U. T. C.).

Moderately accurate station timing procedures as mentioned in section 3 are necessary if a camera is fixed in orientation during the satellite exposure. Since the stellar system is moving, the stellar system must be fixed by an accurate sidereal time (i. e., accurate with respect to a time reference of the satellite).

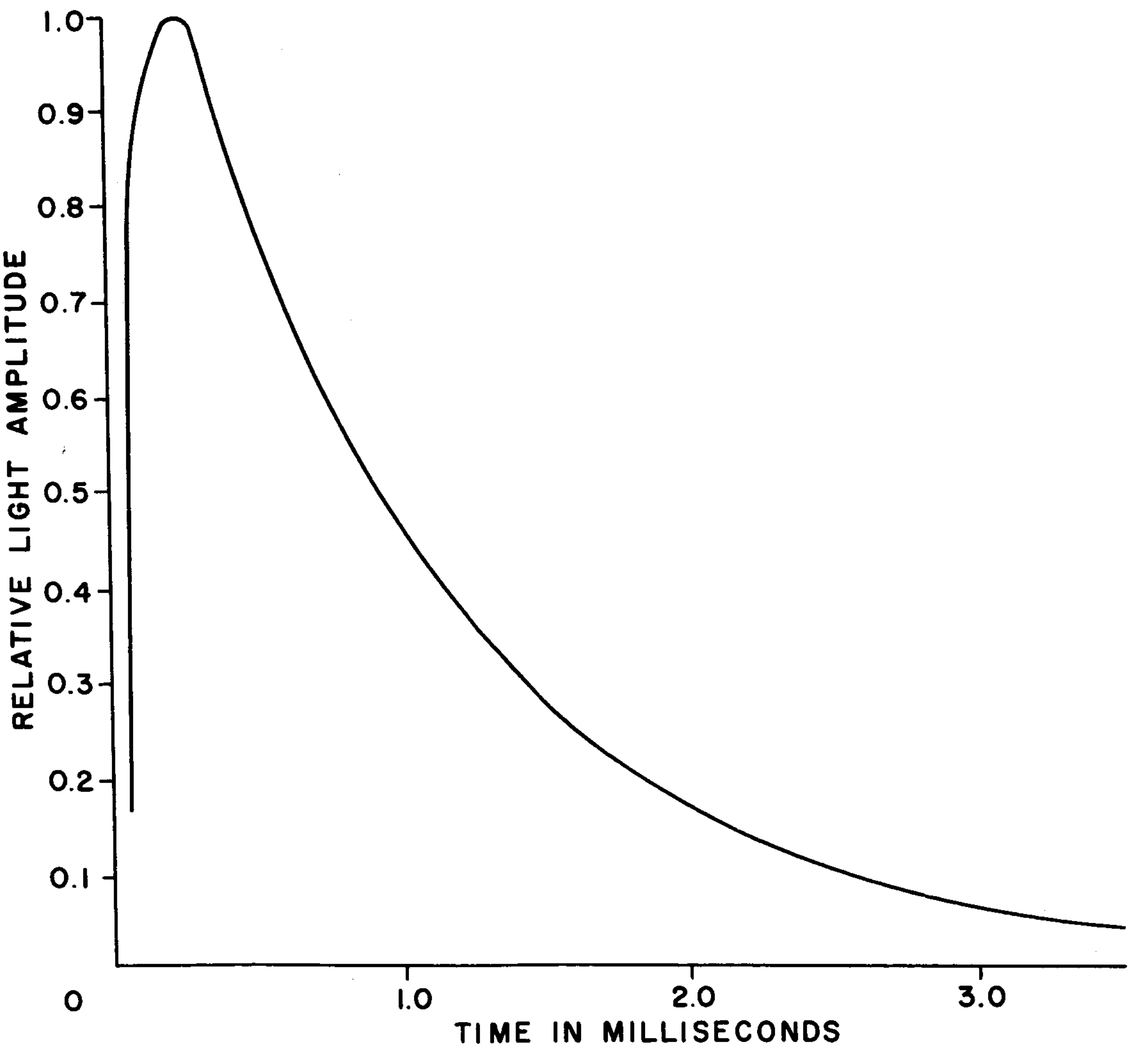
Accurate station timing procedures are not critical, in fact not needed, if a sidereally driven camera is used. In this case the stars remain stationary, and if the right ascension-declination system is used as a reference system for the advanced plate reduction procedure or, if the simplified plate reduction procedure is used, the middle or average flash (over a five second interval) is certainly suitable for updating the stars to their "observed" position.

3.32 Passive Satellite Timing Procedures

Timing in the passive case is, of course, extremely critical. If the camera is sidereally driven and the satellite image is "chopped," the epochs of satellite observation will be determined by the shutter action. If the camera

FIGURE 7

LIGHT OUTPUT VERSUS TIME FOR GEOS-A



is fixed the epoch of star and satellite observations are determined by the shutter action.

In passive satellite observations, the epochs of observation refer to the time the satellite is exposed at the camera (i. e., the station time). As mentioned in section 3.211, the station time of satellite observation should be referred to the time the light left the satellite so simultaneous observations can be determined and difficult "satellite aberration" corrections may be avoided.

4 SPECIFIC AGENCY PROCEDURES

4.1 INTRODUCTION

The previous two sections have briefly outlined how the stellar system may be used, correction necessary to refer the satellite to its true geometric position, plate reduction procedures, and timing procedures. The purpose of this section is to outline the method in which the general concepts in Sections 2 and 3 are applied by various agencies.

The techniques outlined in this section are the result of conversations with the persons listed in the acknowledgements to this report. The reader must realize that the brief outline of procedures contained in this section may reflect many years of research by the individual agency and many of the techniques or procedures have not been published. Therefore, the following procedures should be regarded as preliminary though they have been reviewed by the agencies.

The following abbreviations are used throughout this section. The Minitrack Optical Tracking System operated by the National Aeronautical and Space Administration is designated the MOTS system operated by NASA; The Aeronautical Chart and Information Center is designated ACIC; the Smithsonian Astrophysical Observatory is designated SAO; and the U. S. Department of Commerce Environmental Science Services Adminstration (Coast and Geodetic Survey, and Institute for Earth Sciences) is designated ESSA.

All of the agencies indicated that the GEOS data output to the Geodetic Satellite Data Center (GSDC) has been in the format outlined in Appendix 1 to this report [Kahler et al., 1965, pp. 24-33].

4.2 MINITRACK OPTICAL TRACKING SYSTEM PROCEDURES

4.21 General

The Minitrack Optical Tracking System (MOTS) consists of twelve MOTS cameras located at Space Tracking and Data Acquisition Network (STADAN) sites where the cameras have been used for calibration of electronic Minitrack Interferometers. Nine additional MOTS and PTH-100 cameras have been used in observing the GEOS-A Satellite at selected locations in the Eastern United States [Moss et al., 1966, p. 4].

The MOTS cameras are equatorially mounted, sidereally driven, and have a focal length of 610 mm (MOTS 24), and 1016 mm (MOTS 40) with apertures of 102 mm and 203 mm, respectively. The accuracy of the MOTS sidereal drive has been questioned. An analysis of the sidereal drive system has been accomplished by individuals from the Radio Corporation of America (RCA) and the analysis indicates that periodic errors of 0.5 seconds of arc can occur due to imperfections in the drive mechanism [Brown Associates Inc., 1966, p. 3].

The PTH-100 is a new camera system currently being developed for satellite observations. At the present time three PTH-100 cameras are being used. The PTH-100 is operated in the fixed mode and has the same aperture and focal length as the MOTS-40.

The cameras use 8 × 10 inch glass plates and the field of view is $18° \times 23°$, $11° \times 14°$, and $10° \times 10°$ for the MOTS 24, MOTS 40 and PTH-100, respectively.

The accuracy with which the time of shutter action can be related to radio time signals received at the station is estimated to be 0.75 seconds for the MOTS cameras and 25 milliseconds for the PTH-100 camera. A new shutter system for the MOTS cameras is being investigated [Moss et al., 1966, p. 7]. However, the new shutter system was not used for GEOS-A observations.

The MOTS 24, MOTS 40, and PTH-100 cameras have internal shutters. The MOTS shutter timing accuracy is immaterial since the cameras are sidereally driven and, for geodetic purposes, only observe active satellites (i.e., GEOS-A). The PTH-100 has an internal shutter system of moderate precision. The timing accuracy of a PTH-100 shutter action relative to radio time signals received at the observing station is estimated to be within 25 ms. For geodetic purposes, the PTH-100 is also used exclusively for active satellite observations.

A new shutter system with an exposure time capability of five milliseconds is currently being investigated for the MOTS cameras. However, the new shutter is not currently in use [Moss et al., 1966, p. 7].

4.22 Timing Reduction Procedures

For the active satellite GEOS-A flash times published by the APL are used and are corrected by use of the APL Bulletins (see Section 3.31). The epoch of flash observation corresponds to the time the satellite flash was triggered.

The time used to determine the star image positions (for the PTH-100) are obtained as follows. The PTH-100 stations are equipped with Brush Timing Recorders. WWV radio signals are received and recorded on one channel of the Brush Recorder and the shutter action is recorded on another channel. The instant the shutter is fully opened and starts to close is recorded and compared to the received WWV signal to determine the epoch of shutter opening. The three PTH-100 stations (for GEOS-A observations) were located in Maryland, Indiana, and Florida; consequently, time propogation delays were assumed to be less than 8 ms and were ignored.

4.23 Star Updating Procedures

The SAO Catalogue is used exclusively for MOTS plate reduction. The catalogued star positions are updated to the apparent position and corrected for diurnal aberration as outlined in the following paragraphs.

Proper motion corrections for the epoch of observation are applied directly to the catalogued positions with the following formulas:

$$\begin{aligned} \alpha_1 &= \alpha + \left(\mu_\alpha + \frac{1}{2}\,\Delta\mu_\alpha\right)\Delta T \quad , \\ \delta_1 &= \delta + \left(\mu_\delta + \frac{1}{2}\,\Delta\mu_\delta\right)\Delta T \quad , \end{aligned} \tag{4.1}$$

where μ_α and μ_δ are the catalogued values of proper motion in right ascension and declination and $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ are computed as outlined in [Smart, 1962]:

$$\begin{aligned} \Delta\mu_\alpha &= 2\mu_\alpha\,\mu_{\delta_o}\tan\delta_o\;\;\Delta T, \\ \Delta\mu_\delta &= -\mu_\alpha^2\sin\delta\cos\delta\;\;\Delta T\;. \end{aligned} \tag{4.2}$$

The time in tropical years from the catalogue epoch ($T_\circ$) of 1950.0 is found from

$$\Delta T = T + t - T_\circ \quad , \tag{4.3}$$

where T is the beginning of the nearest Besselian year, and t is the fraction of a tropical year to T. The value of t is taken from the Nautical Almanac for the current year. During the last half of the year the value of t will be negative since the tabulated value of t refers to the beginning of the next Besselian year.

The star coordinates are then precessed by using approximate expressions for the equations given in equations 2.15. The formulas used are,

$$\begin{aligned} \zeta_\circ &= [23''.04948 + 0.00014\,(T_\circ - 1950)]\,\Delta T + 3''.0 \\ &\quad \times 10^{-5}\,\Delta T^2 + 1''.7 \times 10^{-8}\,\Delta T^3 \quad , \\ \theta &= [20''.04255 - 0.000085\,(T_\circ - 1950)]\,\Delta T - 4''.3 \\ &\quad \times 10^{-5}\,\Delta T^2 - 4.1 \times 10^{-8}\,\Delta T^3 \quad , \\ z &= \zeta_\circ + 7.9 \times 10^{-5}\,\Delta T^2 \quad , \end{aligned} \tag{4.4}$$

where $T_\circ$ is the epoch of the catalogue, ΔT is the interval in years from $T_\circ$ to the epoch of observation computed as indicated in the previous paragraph. These equations are a simplification of equation 2.15. The corrections are applied as outlined in Section 2.123.

The next step is to apply nutation for the epoch of observation to the precessed coordinates. The nutation in longitude $\Delta\psi$ is taken from the

"Nutation in Longitude" column tables of the "Sun, 196X" tables in the current Nautical Almanac. The true obliquity of the ecliptic (ϵ) is taken from the tenth column of the same table. The mean obliquity ϵ_m of the ecliptic is found from

$$\begin{aligned} \epsilon_m = {} & 23.445787 - 1.30125 \times 10^{-4} \Delta T \\ & - 1.64 \times 10^{-10} \Delta T^2 \quad . \end{aligned} \tag{4.5}$$

The nutation in obliquity is then determined from

$$\Delta\epsilon = \epsilon - \epsilon_m \quad . \tag{4.6}$$

The nutation is then applied to the precessed coordinates α_p, δ_p by the following expressions:

$$\begin{aligned} \delta &= \delta_p + \arcsin(\Delta\epsilon \sin\alpha_p + \Delta\psi \cos\alpha_p \sin\epsilon_m), \\ \sin\alpha &= (\sin\alpha_p \cos\delta_p - \Delta\epsilon \sin\delta_p + \Delta\psi \cos\epsilon_m \cos\delta_p \cos\alpha_p)/\cos\delta, \\ \cos\alpha &= [\cos\alpha_p \cos\delta_p - \Delta\psi(\sin\delta_p \sin\epsilon_m + \cos\delta_p \cos\epsilon_m \sin\alpha_p)]/\cos\delta \ . \end{aligned} \tag{4.7}$$

The next step is to correct the coordinates for annual aberration. The mean longitude of the Sun and the correction to the apparent longitude are linearly interpolated from the "Sun, 196X" tables in the current Nautical Almanac. The apparent longitude of the Sun ($\odot$) is then found by adding the correction to the mean longitude. The correction for annual aberration is then applied as basically outlined in Smart [Smart, 1962, pp. 182-183]. The annual aberration constant K is set equal to 20".47 and the following corrections are used:

$$\delta = \delta' + \arcsin\left[-K\cos\odot\cos\epsilon_m(\tan\epsilon_m\cos\delta' - \sin\alpha'\sin\delta') - K\cos\alpha'\sin\delta'\sin\odot\right], \tag{4.8}$$

$$\alpha = \alpha' + \arcsin\left[(-K\cos\alpha'\cos\odot\cos\epsilon_m - K\sin\alpha'\sin\odot)/\cos\delta\right],$$

where α', δ' are the coordinates before correction, and α, δ are the corrected coordinates.

The diurnal aberration correction is then applied. The hour angle (h) is computed as outlined later is Section 4.24 and the correction is applied using the following formula:

$$\begin{aligned}\delta &= \delta' + \arcsin(\bar{D}\sin\delta\sin h), \\ \alpha &= \alpha' + \arcsin(\bar{D}\cos h/\cos\delta),\end{aligned} \tag{4.9}$$

where the primed and unprimed right ascensions denote the uncorrected and corrected coordinates, respectively, and $\bar{D}$ is equal to $0\overset{''}{.}319\cos\varphi$ where φ is the station latitude. These corrections are equivalent to those given in equation 2.29. Astronomic refraction is applied to the star coordinates during plate reduction.

4.24 Plate Reduction Procedures

All GEOS-A plates and future plates are reduced at the New Mexico State University using a plate reduction technique similar to the procedure outlined in Section 3.2.

Presently, the plate reduction procedure uses predetermined lens distortion coefficients. To determine the coefficients a calibration plate is exposed to determine the lens distortion parameters. The plates used for satellite observations are then reduced by using the distortion constants determined from the plate calibration. Recent results have indicated that the procedure may not be adequate for the MOTS cameras since the camera may be refocused (hence the distortion coefficients changed) between the camera calibration exposure and satellite exposure. Currently, a program is in progress to recompute the distortion coefficients for plates that have had "shifts" in the x_p and y_p coordinates. The Duane Brown Associates, Inc. will compute distortion values for the questionable plates and forward the calibration values to the New Mexico State University. The New Mexico State University will then redetermine the flash direction [Brown Associates, 1966, pp. 5-6].

Approximately 40 to 50 star images near the satellite images are chosen as reference points for plate reduction. For the MOTS cameras, star magnitudes of 7.5 to 9.0 are generally used, since stars of greater magnitude are generally not catalogued and stars of lesser magnitude have larger images than the flash images. The stars chosen are identified by matching the plate with a star chart which is the same "scale" as the plate.

The plate is then measured on a Model 422D Mann Comparator which is equipped with a device to automatically punch the measurements on cards. The measurements are referenced to a nominal plate center determined by fiducial marks or, in the absence of fiducial marks, by the intersection of lines from the plate corners. Each star or flash image is measured five times and the average of the best four measurements of the five are used in the plate reduction.

Lens distortion corrections are applied to the measured plate coordinates. Equation 3.24 is used to apply radial lens distortion. The following formula is used to compute the tangential or decentering distortion:

$$\begin{aligned} \Delta x &= [P_1(r^2 + 2x^2) + 2P_2\, x\, y][1 + P_3\, r^2] , \\ \Delta y &= [2P_1\, x\, y + P_2(r^2 + 2y^2)][1 + P_3 r^2] , \end{aligned} \tag{4.10}$$

where P_1, P_2, and P_3 are the predetermined coefficients in this case (instead) of J_1, J_2 in equation 3.25).

The star coordinates are updated to the apparent place using procedures outlined in Section 4.23. Refraction corrections are then applied to each star image by the Garfinkel method. In this case, the auxiliary angle $\bar{\theta}$ is computed from equation 2.34 and γ_o is computed from equation 2.35. The coefficients $\eta_1, \eta_2, \ldots, \eta_5$ are determined from:

$$\begin{aligned} \eta_1 &= a \\ \eta_2 &= a(2/3 + b) \\ \eta_3 &= a(2/7 + 17/11\, b + 2b^2) \\ \eta_4 &= a(1/14 + 87/77\, b + 185/44\, b^2 + 5\, b^3) \\ \eta_5 &= a(1/126 + 70/143\, b + 3155/748\, b^2 + 815/66\, b^3 + 14 b^4) \end{aligned} \tag{4.11}$$

where

$$\begin{aligned} a &= 2\, d\gamma_o(1 + d) , \\ b &= 2d\, \gamma_o , \end{aligned} \tag{4.12}$$

and d is a function of the index of refraction u at the observer

$$d = \frac{\hat{u}^2 - 1}{2\hat{u}^2} \quad . \tag{4.13}$$

The index of refraction $\hat{u}$ is dependent on the pressure p_o in millibars, the temperature T_o in degrees Kelvin, and the wave length $\hat{\tau}_o$ in microns of the light observed

$$\hat{u} = 1 + [(77.34 + 0.44/\hat{\tau}_o^2)P_o/T_o] \times 10^{-6} \quad . \tag{4.14}$$

In the plate reduction $\hat{\tau}_o$ is assumed to be 0.54 microns and a and b are constrained parameters.

The general projective equations are then solved as outlined in Section 3.23. The adjustable parameters are ψ°, ω°, $\varkappa^\circ$, $\bar{x}_p^\circ$, $\bar{y}_p^\circ$, c°, a° and b° (the zero superscripts denote the approximate values of the adjustable parameters). The approximate values of ψ°, ω°, $\varkappa^\circ$ are computed from

$$\psi^\circ = \tan^{-1}\left[\frac{-\cos\delta^\circ \sin h}{\sin\delta^\circ \cos\varphi - \cos\delta^\circ \sin\varphi \cos h}\right] ,$$

$$\omega^\circ = \sin^{-1}[\sin\delta^\circ \sin\varphi + \cos\delta^\circ \cos\varphi \cos h] , \tag{4.15}$$

$$\varkappa^\circ = \cos^{-1}[\cos h \cos\psi^\circ - \sin h \sin\varphi \sin\psi^\circ] ,$$

where α°, δ° are the estimated right ascension and declination of the plate center; φ, λ are the latitude and longitude of the camera station, and h is the hour angle computed from

$$h = \theta_o + (1.0027379167)\ UT(hrs) - \lambda - \alpha^\circ , \tag{4.16}$$

where θ_o denotes the apparent sidereal time at Greenwich at 0^hU. T. and is taken from the current Nautical Almanac for the date of observations; U. T. (hrs) is the U. T. interval from zero hours U. T. to the epoch of observation, and λ is the station longitude (positive when west). The precise values of φ and λ used in plate reduction are determined at the New Mexico State University; however, the output directions to the satellite are in the right ascension-declination system so the precise values of φ and λ used are not required to use the output data.

The projective equations are reiterated until the following three conditions are met, or until ten iterations have been completed:

1. The adjustable angular quantities in the n and n-1 iterations differ by less than 0.5 seconds of arc.

2. The center point $\bar{x}_p$, $\bar{y}_p$ values agree within two microns in the n and n-1 iterations.

3. The residual vectors of an image point agree within four microns in the n and n-1 iterations.

If ten iterations do not produce the above conditions the residual vectors are examined. If any vector component exceeds some value, set to 20 microns for good quality plates, the star showing the greatest residual is rejected and the plate reduction is re-run. This procedure is repeated until all residuals are $\leq 20\,\mu$. If it is required to reject more than 9 stars the reduction is halted and the data are re-examined for possible errors or for generally poor plate quality.

The adjusted orientation parameters are then used to determine the refracted direction to the ray in the azimuth-altitude system by use of equation 3.10. Equation 3.10, however, assumed a "fixed" camera system; therefore, for the MOTS sidereally driven system, the orientation elements only refer to one particular epoch defined by a specific flash time. If we designate the specific flash time t_s, the orientation elements for any other flash time t_1 are found by use of the following formulas:

$$\begin{aligned}
\psi_1 &= \psi_o + (\sin\varphi - \tan\omega_o \cos\psi_o \cos\varphi)(t_1 - t_s) \quad, \\
\omega_1 &= \omega_o + (\sin\psi_o \cos\varphi)(t_1 - t_s) \quad, \\
\chi_1 &= \chi_o + \csc\chi_o \, [-\sin h_o \cos\psi_o + \cos h_o \sin\varphi \sin\psi_o - (\sin\varphi \\
&\quad -\tan\omega_o \cos\psi_o \cos\varphi)(\cos h_o \sin\psi_o - \sin h_o \sin\varphi \cos\psi_o)]\,(t_1 - t_s) \quad,
\end{aligned} \tag{4.17}$$

where the zero subscript refers to the orientation elements at time t_s, φ is the station latitude, and h_o is the hour angle at the time t_s. With the above equations solved for a particular flash time t_1, equation 3.10 can be used to find the azimuth and refracted altitude to any flash image.

After the refracted satellite altitude is found, corrections are applied for astronomical and parallactic refraction. If we denote the refracted altitude by ω and the unrefracted altitude by $\overline{\omega}$, we can solve equation 3.15 for $\overline{\omega}$ where

$$\Delta\overline{\omega} = \omega - \overline{\omega} \quad . \tag{4.18}$$

The value of $\Delta\overline{\omega}$ is obtained directly from the Garfinkel formula (equations 2.33-2.35) using the refracted zenith distance $(\pi/2 - \omega)$ to determine θ and computing the coefficients η_1, η_2, etc., in the basic Garfinkel expression using the "adjusted" values of a and b in equation 4.11. The elevation of the ray corrected for astronomic refraction and parallactic refraction $(\overline{\overline{\omega}})$ is given by

$$\overline{\overline{\omega}} = \cot^{-1} \frac{\sin\beta}{\cos\beta - f_R} \quad . \tag{4.19}$$

Where

$$f_R = \frac{R_o + h_s}{R_o + h_o} \quad , \tag{4.19a}$$

and

$$\beta = \frac{\pi}{2} - \omega - \Delta\overline{\overline{\omega}} - \sin^{-1}\left[\; u\, f_R \sin\left(\frac{\pi}{2} - \omega\right)\right] , \tag{4.20}$$

in which

R_o = Radius of curvature of the meridian. R_o can be calculated by classic geodetic means using the station coordinates.

h_s = Height of the station above mean sea level.

h_o = Height of the satellite above mean sea level.

u = Index of refraction at the camera station (equation 4. 14)

A complete explanation of the above technique is given by Brown [Brown, 1957, pp. 33-51].

If a MOTS plate is reduced, the corrections for refraction are computed using a specific flash (at epoch t_s) azimuth and altitude. Consequently, a correction is added to a flash at time t_1 in the amount

$$\Delta\overline{\overline{\omega}}' = (0.3 \times 10^{-3} \sin \overline{\overline{\psi}} \cos \varphi \operatorname{cosec}^2 \overline{\overline{\omega}})\,(t_1 - t_s) \quad , \tag{4.21}$$

where $\overline{\overline{\psi}}$ denotes the azimuth of the ray found from the basic projective equations and φ, $\overline{\overline{\omega}}$, t_1, t_s are defined in the preceding paragraphs.

The azimuth and elevation ($\overline{\overline{\psi}}$ and $\overline{\overline{\omega}}$) of a given ray are then converted to right ascension (α) and declination (δ) by using the following formulas:

$$
\begin{aligned}
\sin\delta &= \sin\varphi\sin\bar{\bar{\omega}} + \cos\varphi\cos\bar{\bar{\omega}}\cos\bar{\bar{\psi}} \quad , \\
\sin h &= -\frac{\sin\bar{\bar{\psi}}\cos\bar{\bar{\omega}}}{\cos\delta} \quad , \\
\cos h &= \frac{\cos(\pi/2 - \bar{\bar{\omega}}) - \sin\varphi\sin\delta}{\cos\varphi\cos\delta} \quad , \\
\alpha &= \theta_\circ + (1.0027379167)\ \mathrm{UT\ (hrs)} - h - \lambda,
\end{aligned}
\tag{4.22}
$$

where $\theta_\circ$ and U. T. (hrs) are defined in equation 4.16, and λ is the observer's longitude, positive west.

Estimates of the covariance $\sigma_{\alpha\delta}$ and standard deviations σ_α and σ_δ of the right ascension and declination of star images are estimated by assuming the flashes are in error by the amount of the average standard error of a star residual. The right ascension-declination covariance matrix is given by

$$
\underset{\sim}{\Sigma}_{\alpha\delta} = \begin{pmatrix} \sigma_\alpha^2 & \sigma_{\alpha\delta} \\ \sigma_{\alpha\delta} & \sigma_\delta^2 \end{pmatrix} = \bar{\underset{\sim}{B}}\ \underset{\sim}{G}\ \bar{\underset{\sim}{B}}^T \quad , \tag{4.23}
$$

where

$$
\bar{\underset{\sim}{B}} = \begin{pmatrix} \dfrac{\partial\alpha}{\partial x} & \dfrac{\partial\alpha}{\partial y} \\[2ex] \dfrac{\partial\delta}{\partial x} & \dfrac{\partial\delta}{\partial y} \end{pmatrix} \quad , \tag{4.24}
$$

and $\underset{\sim}{G}$ is the covariance matrix of the image measurements. The terms in the $\bar{\underset{\sim}{B}}$ matrix are approximated by

$$
\begin{aligned}
\frac{\partial\alpha}{\partial x}\cos\delta &= \frac{\alpha_1 - \alpha}{\Delta} & \qquad \frac{\partial\alpha}{\partial y}\cos\delta &= \frac{\alpha_2 - \alpha}{\Delta} \\
\frac{\partial\delta}{\partial x} &= \frac{\delta_1 - \delta}{\Delta} & \qquad \frac{\partial\delta}{\partial y} &= \frac{\delta_2 - \delta}{\Delta}
\end{aligned}
\tag{4.25}
$$

where the subscripts 1 and 2 indicate the values obtained by replacing the actual x, y measurements by $x + \Delta$ and $y + \Delta$, respectively. The Δ term is given a value of 100 microns. This is not the magnitude of a star residual, however.

4.25 Final Results

The final results are forwarded to the Geodetic Satellite Data Center (GSDC). The satellite coordinates are in the right ascension-declination system and represent the true topocentric position referred to the true equator and equinox at the epoch of observation.

Since only active satellites are observed, the time reported corresponds to the U.T.C. time the satellite flash mechanism was triggered.

4.3 SMITHSONIAN ASTROPHYSICAL OBSERVATORY PROCEDURES

4.31 General

The Smithsonian Astrophysical Observatory uses Baker Nunn Cameras for satellite observations. The camera consists of a modified Super Schmidt f/1 Telescope on a triaxial mount which allows operation in a tracking mode, or fixed mode. The triaxial mount allows the camera to track along any great-circle arc with an adjustable angular velocity between 0 and 7000"/sec. Although tracking modes are possible all GEOS-A observations were made in the stationary mode.

The Baker-Nunn Camera has a 50 cm focal length and aperture. Film is used to record the satellite/stellar exposure. The film support is a spherical surface which is characteristic of a Schmidt optical system [Abell, 1964, pp. 146-147]. The field of view of the camera is 30° along the tracking axis, and 5° perpendicular to the tracking axis. After one exposure, the film can be shifted along the tracking axis and another exposure made [Moss et al., 1966, p. 19].

A clam shell capping shutter, which has exposure time settings from 0.2 to 3.2 seconds, is used to begin and terminate the exposure. A barrel (timing) shutter, mounted in front of the spherical focal surface, has two staves and rotates two and one half times giving five breaks during a normal exposure at a highly precise angular velocity. During the third break (of five) an electric contact closes, demanding an illuminated time display inside the camera which appears on the film with the star and satellite images. The photographed time presentation consists of ten digital nixie displays depicting time from 0.1ms to hours.

Currently, a new camera system designated the K-50 is being developed by the SAO. The K-50 has a 100 cm focal length, a 25 cm aperture, and an internal shutter system. Unlike the Baker-Nunn, the K-50 has a flat photographic plate. The accuracy with which the shutter action can be determined is currently being investigated. Three prototype models are presently being tested by the SAO.

4.32 Timing Reduction Procedures

For the active satellite, GEOS-A, flash times published by the APL are corrected by use of the APL Bulletins and a flash delay correction of 0.75 ms is added. The resulting epoch corresponds to the time of maximum brilliance rather than to the time the flash was triggered. After the flash delay correction is applied, a light time correction is added to the flash time (i. e., the time of maximum brilliance) to reduce the epoch to the time the flash was observed at the station. The correction for light time is

$$\Delta t = \frac{r_s}{c} \quad , \tag{4.26}$$

where Δt is the light time correction, r_s is the distance to the satellite in kilometers, and c is the velocity of light (299792.5 km/sec). The value of r_s is computed using the current orbital elements of the satellite. Finally a correction is applied to refer the flash time (in U. T. C.) to an atomic time (A. S.).

The correction A. 1 - UT1 is obtained from the Preliminary Emission Bulletins published by the U. S. Naval Observatory. The U. S. Bureau of Standard's Bulletins are also used to determine the precise frequency offset. of WWV and A. S. Station timing is referenced to the NBS(UA) time scale which controls WWV. Since the WWV emitted signal is controlled by an atomic standard the correction will be of the linear form

$$\text{A. S.} - \text{U. T. C.} = a + b\,(t - t_o)\ldots \quad , \tag{4.27}$$

where a is the initial offset at an arbitrary epoch t_o and b is the linear rate of change (in seconds of time per $t - t_o$) of A. S. frequency with respect to the WWV frequency. The values of a will change if the U. T. C. signal is "step adjusted" to more closely agree with U. T. 2 and the value of b will change if a frequency adjustment of the U. T. C. time is made.

TABLE 3
SAO ADOPTED VALUES FOR A. S. - WWV (emitted)

DATES	A. S. - WWV (emitted)
1961 Jul 01.0 - 1961 Jul 13.0	1.693 434 + 0.001 292 000 (t - 37,480.0)
1961 Jul 13.0 - 1961 Aug 01.0	1.694 215 + 0.001 245 000 (t - 37,480.0)
1961 Aug 01.0 - 1961 Sep 21.0	1.643 160 + 0.001 280 000 (t - 37,480.0)
1961 Sep 21.0 - 1961 Oct 01.0	1.641 500 + 0.001 300 000 (t - 37,480.0)
1961 Oct 01.0 - 1961 Nov 01.0	1.642 184 + 0.001 290 764 (t - 37,480.0)
1961 Nov 01.0 - 1962 Jan 01.0	1.643 272 + 0.001 289 444 (t - 37,480.0)
1962 Jan 01.0 - 1962 Apr 01.0	1.865 000 + 0.001 123 200 (t - 37,650.0)
1962 Apr 01.0 - 1962 Jul 01.0	1.864 620 + 0.001 126 800 (t - 37,650.0)
1962 Jul 01.0 - 1963 Jan 01.0	1.864 704 + 0.001 126 370 (t - 37,650.0)
1963 Jan 01.0 - 1963 Nov 01.0	2.292 725 + 0.001 118 458 (t - 38,030.0)
1963 Nov 01.0 - 1964 Jan 01.0	2.392 725 + 0.001 118 458 (t - 38,030.0)
1964 Jan 01.0 - 1964 Apr 01.0	2.800 962 + 0.001 293 560 (t - 38,395.0)
1964 Apr 01.1 - 1964 Jul 01.0	2.900 766 + 0.001 295 716 (t - 38,395.0)
1964 Jul 01.0 - 1964 Sep 01.0	2.901 518 + 0.001 292 659 (t - 38,395.0)
1964 Sep 01.0 - 1964 Oct 01.0	3.001 518 + 0.001 292 659 (t - 38,395.0)
1964 Oct 01.0 - 1965 Jan 01.0	3.001 589 + 0.001 296 048 (t - 38,395.0)
1965 Jan 01.0 - 1965 Mar 01.0	3.575 732 + 0.001 296 000 (t - 38,761.0)
1965 Mar 01.0 - 1965 Jul 01.0	3.675 732 + 0.001 296 000 (t - 38,761.0)
1965 Jul 01.0 - 1965 Sep 01.0	3.775 732 + 0.001 296 000 (t - 38,761.0)
1965 Sep 01.0 - 1966 Jan 01.0	3.875 732 + 0.001 296 000 (t - 38,761.0)
1966 Jan 01.0 - 1966 May 01.0	4.348 772 + 0.002 592 000 (t - 39,126.0)

The adopted value of the A. S. -WWV constants presently used by the SAO are given in Table 3 as a function of the date of use. The value of t in the tabulated expressions must be expressed in Modified Julian Days. The Modified Julian Day of observation is the Julian Day of observation minus 2400000.5 days [Veis, 1963, p. 5]. The epoch of observation of a flashing satellite, therefore, refers to the flash (i. e., the instant of maximum brilliance) time at the camera station and is expressed in A. S.

The station timing system is used in observing passive satellites and was used in orienting the stellar system during the GEOS-A observations. The station timing system consists of a master clock (a quartz crystal clock) that is used as a timing standard at the station and a slave clock attached to the camera [Moss et al., 1966, p. 20]. In the past, the master clock was referenced to UTC by monitoring radio time signals and applying propagation corrections as outlined previously; however, at present the master clocks are set with a portable crystal clock and maintained by VLF transmissions. The portable crystal clock is set to NBS (UA) time scale in Boulder, Colorado. In December 1966, eight master station clocks had been set by the portable clock method. The stations involved and the estimated total station timing uncertainty to NBS (UA) are given in Table 4. Setting uncertainty is generally below ± 100 μs.

TABLE 4
SAO STATIONS THAT HAVE HAD THE STATION EPOCH CHECKED WITH A PORTABLE CLOCK*

STATION NO.	LOCATION	DATE	TOTAL UNCERTAINTY (in microseconds)
9001	New Mexico	8 Dec 65	170
9002	South Africa	19 Nov 66	320
9004	Spain	5 Nov 66	235
9005	Japan	31 Oct 66	400
9007	Peru	5 Dec 65	265
9010	Florida	27 Oct 66	85
9012	Hawaii	6 Oct 66	130
9023	Australia	28 Aug 66	308

* As of December 1966

The time recorded on the photograph plate receives basically the following corrections: First, since the shutter rotates with a slow but highly precise angular velocity, a sweep shutter correction must be applied to reduce the time of observation from the center of the frame to the position of the satellite on the frame [Moss et al., 1966, p. 23]. Second, the time of the master station clock (U.T.C.) is corrected to A.S. in the same manner as outlined above for active satellite observations.

4.33 Star Updating Procedures

The SAO Catalog is used for all SAO plate reductions, and star positions used in the plate reduction are referred to the equator and equinox of 1950.0 and an epoch never more than 0.5 years from the epoch of observation. Proper motion is applied to the catalogued coordinates using the following formulas:

$$\begin{aligned} \alpha &= \alpha' + \mu_\alpha \Delta T \quad , \\ \delta &= \delta' + \mu_\delta \Delta T \quad , \end{aligned} \tag{4.28}$$

where α', δ' are the catalogued right ascension and declination μ_{α}, μ_δ are the catalogued proper motions, and ΔT is time in tropical years from 1950.0 to an epoch never more than 0.5 years away from the epoch of observation, e.g., for observations in 1966, the updated epoch is 1966.5.

A final correction for annual aberration is applied to the satellite image after the satellite position has been determined by the plate reduction. The annual aberration correction will be described in the next section.

4.34 Plate Reduction Procedures

The SAO films are reduced using the linear plate constant or "Turner's" method described in Section 3.24. Since the mean positions of stars are used, corrections for differential effects (precession, nutation, etc.) between the mean and observed standard coordinates of stars are assumed to be included in plate constants.

A computer program is used to compute the approximate satellite position, select a set of bright stars for identification purposes in the vicinity of the satellite position, and select a set of stars symmetrically distributed about the satellite image for measurement. Finally, the person measuring the plate selects six to eight symmetrically distributed stars based on individual image quality and distance from the satellite image. If possible, the stars selected are within 20 mm (2 degrees) of the satellite image [Moss et al., 1966, p. 21]. A new reduction program presently being instituted eliminates this pre-reduction program, allowing the measurer to select his stars at the measuring engine, the coordinates being selected by the reduction program.

The star and satellite image measurement is then performed on one of five Mann Comparators (the SAO currently has one model 422D and four model 829 A's) that are calibrated yearly. The comparators are equipped with a projection system which projects the images to be measured and the comparator cross hairs onto a screen. Measurements of the star and satellite images are made in sets of three "pointings" on each image ($3\bar{x}$ and $3\bar{y}$ measurements in each set). The satellite image is measured three times, each star image is measured three times, and the satellite image is again measured three times. The film is rotated 180° and the same sequence is repeated. Finally, two points on the edge of the film and the projected images of the shutter calibrator are each measured once. These measurements are used to compute the "sweep shutter" and the "shutter flutter" corrections, which correct the photographed time for the time required for the shutter to rotate from the position where it intercepted the satellite image to the position it had when the clock was photographed, and correct for imperfect centering of the sweep shutter.

The star positions are then converted to the standard coordinate system by use of equation 3.53 given in Section 3.24. The origin of the standard coordinate system has in the past been determined by the following formulas:

$$X_o = \frac{\sum_{i=1}^{n} X_i}{n} ,$$

$$Y_o = \frac{\sum_{i=1}^{n} Y_i}{n} , \qquad (4.29)$$

$$Z_o = \frac{\sum_{i=1}^{n} Z_i}{n} ,$$

where X_o, Y_o, Z_o denote the direction cosines of the average star coordinate in the right ascension-declination system; X_i, Y_i, Z_i denote the direction cosines of the i^{th} star images in the same system; and n is the number of stars. The origin of the standard coordinate system in the right ascension-declination system (α_o, δ_o) may be determined by equation 2.28, but this is not in fact done by the reduction program.

The standard coordinates and measured coordinates for each star are used to generate the linear plate constant equations of the form

$$\begin{aligned} \xi &= a\bar{x} + b\bar{y} + c' \quad , \\ \eta &= a'\bar{x} + b'\bar{y} + c'' \quad , \end{aligned} \tag{4.30}$$

where $\bar{x}$ and $\bar{y}$ are obtained (for each star image) from

$$\bar{x} = \frac{(\underline{x} - x_s) + (\underline{x}' - x_s')}{2} \quad , \qquad \bar{y} = \frac{(\underline{y} - y_s) + (\underline{y}' - y_s')}{2} \quad . \tag{4.31}$$

In the above equations $\underline{x}$, $\underline{y}$ are the average values of the measured star coordinates in the direct measurement; $\underline{x}'$, $\underline{y}'$ are the average values for the direct satellite image measurements; x_s, y_s are the average values of the direct satellite measurement; and x_s', y_s' are the average values of the reverse satellite measurement. The averaged coordinates are examined and a star is rejected in a measurement differs by more than about 20 microns from the mean, or the direct and reverse measurements do not coincide. If more than two stars are rejected, the plate is remeasured. The above formulas are used to place the satellite image at the center of the arbitrary x, y coordinate system ($\bar{x}_s = 0$, $\bar{y}_s = 0$).

The linear plate constants are then determined by using the method of least squares. The residual of each star coordinate resulting from the adjustment is computed by the formulas in the next paragraph, and if the standard deviation (of the star residuals) exceeds 25 microns, a star with a large residual is rejected and the adjustment reaccomplished. The final solution must be made with six well-distributed reference stars.

Standard deviations for the plate adjustment are then computed from each star's $\underline{\alpha}$, $\underline{\delta}$ residuals as follows:

$$V_{\alpha}^2 = \frac{\sum_{i=1}^{n} v_{\alpha i}^2}{n - 3} , \qquad V_{\delta}^2 = \frac{\sum_{i=1}^{n} v_{\delta i}^2}{n - 3} , \tag{4.32}$$

where n is the number of stars used in the solution; $v_{\alpha i}^2$, $v_{\delta i}^2$ are the individual star residuals; and V_{α}^2, V_{δ}^2 represent the mean standard errors of the adjustment.

The final adjusted values of the plate constants are then used to determine the standard coordinates of the satellite. Since the measured satellite coordinates determined the origin of the $\overline{x}$, $\overline{y}$ system, the standard coordinates of the satellite image are

$$\begin{aligned} \xi_s &= c' , \\ \eta_s &= c'' . \end{aligned} \tag{4.33}$$

The right ascension (α_s) and declination (δ_s) of the satellite is then found by use of equation 3.66.

The satellite's right ascension and declination are then corrected for the effect of annual aberration. The correction used is applied in the following manner:

$$\begin{aligned} \alpha_s^a &= \alpha_s + C_s c^{*} + D_s d^{*} , \\ \delta_s^a &= \delta_s + C_s c^{**} + D_s d^{**} , \end{aligned} \tag{4.34}$$

where α_s^a, δ_s^a are the satellite coordinates corrected for aberration, and

$$
\left.
\begin{aligned}
c^* &= \cos \alpha_s \sec \delta_s \ , \\
d^* &= \sin \alpha_s \sec \delta_s \ , \\
c^{**} &= \tan \epsilon_o \cos \delta_s - \sin \alpha_s \sin \delta_s \ , \\
d^{**} &= \cos \alpha_s \sin \delta_s \ , \\
C_s &= -K \cos \epsilon_o \cos \bar{\odot} \ , \\
D_s &= -K \sin \bar{\odot} \ .
\end{aligned}
\right\} \tag{4.35}
$$

The author has given physical interpretation to the constants above for use in Section 5. Currently, the above terms are computed from the following expressions (in radians):

$$
\begin{aligned}
\tan \epsilon_o &= 0.4336661 \ , \\
K &= +0.000099241 , \\
K \cos \epsilon_o &= +0.00009104766 \ , \\
\bar{\odot} &= 0.000350811 \sin 2M \\
&\quad + 0.033502133 \sin M + TK_2 + 4.87221261 , \\
TK_2 &= (MJD - 36203.0) \ 0.017202798 , \\
M &= TK_2 + 6.22976345 , \\
MJD &= \text{Modified Julian Date}
\end{aligned}
\tag{4.36}
$$

The above corrections yield the "final" right ascension and declination for a passive satellite regardless of the camera's mode of operation. Correction for parallactic refraction, diurnal aberration, and parallactic aberration are not applied to the satellite image.

As mentioned earlier, the active GEOS-A Satellite was observed with the camera in a fixed mode, therefore, the stellar orientation changed between the time the star images were chopped by the shutter (i. e., the time recorded on the film) and the time the flash occured. The correction applied for the diurnal motion of the sky in seconds of arc is

$$\Delta\alpha = 1.002738 \, (t^* - t_s) \tag{4.37}$$

where t^* is the time at which the measured star images were photographed, and t_s is the station time of satellite flash.

Currently, the standard deviations of right ascension-declination and time that are sent to the Geodetic Satellite Data Service do not result from the plate adjustment, they are arbitrarily set at ±4'' and ±1 ms.

4.35 Final Results

The above procedures complete the data processing procedures; however, the output data is checked in the following manner [Moss et al., 1966, p. 25]. First, the results of a simultaneous observation are subjected to a quadratic polynomial fit. If a deviation greater than four seconds of arc is found, the measurement is discarded. Second, all of the data collected for a satellite during a one-month period is fitted to a "Differential Orbit Improvement" program. If deviation in right ascension and declination exceed three times the standard deviation of the run, an attempt is made to find an error in the plate reduction process. If an error cannot be found and corrected, the observation is rejected.

The estimated and published position error of the satellite (after "Differential Orbit Improvement") is four seconds of arc in right ascension and declination. The standard error of the epoch of observation is estimated to be one millisecond.

The data output consists of the geocentric (i. e., not corrected for diurnal aberration) right ascension and declination of the satelllite image referred to the epoch of observation and to the mean equator and equinox of 1950. 0 with proper motions applied to an epoch never more than 0. 5 year from the epoch of observation. Parallactic refraction and diurnal aberration corrections are not applied to the satellite coordinates. The epoch of observation in the A. S. time system refers to the time of maximum light intensity at the station for active and passive satellites. All additional corrections such as refraction, diurnal aberration, etc., are all explicitly applied to satellite observations when these observations are used for analysis.

4.4 THE ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION PROCEDURES

4.41 General

The Environmental Science Services Administration (ESSA) uses the Wild BC-4 ballistic camera in the fixed mode. The camera has a focal length of 305 mm, an aperture of 117 mm, and a 33° x 33° field of view. The Astrotar lens (with a focal length of 305 mm) is currently used; however, a new lens has been developed and is being installed in the BC-4 cameras. The new lens design was optimized for geodetic satellite triangulation [Schmid, 1966, p. 13].

The shutter mechanism consists of three internal rotating disks and an exterior iris-type shutter which are programmed to open and close with an electronic synchronization system. The auxiliary iris-type shutter is primarily used to chop star trails before and after a satellite pass. The exposure time of the shutter is programmed to produce five well-defined star images in a trail and four trails before and after satellite pass.

The length of time between exposures is determined by the declination of stars in the field of view. During the auxiliary shutter operation the internal shutters are locked in the mid-open position. During a passive satellite pass, the auxiliary shutter is used to reduce the exposure rate and uniquely identify the satellite trail. Two of the internal shutters counter-rotate and constitute the chopping shutter action while the third shutter acts as a chopping disk to limit the exposure rate. The chopping shutters have primary exposure durations of 1/60, 1/30, and 1/5 of a second; however, when the capping shutter is synchronized with the counter-rotating shutters, the exposure rate can be reduced by a ratio of 2 or 5. As mentioned earlier, the auxiliary shutter can be used in conjunction with the capping disk to further limit the exposure rate by a selected factor [Satellite Triangulation in the Coast and Geodetic Survey, 1966, p. 5].

For the active GEOS-A Satellite, the auxiliary iris-type shutter was used to chop the star trails and the shutters were left open for the satellite pass.

The relatively large plate area used and the rapid chopping action of the shutter produce a photogrammetric record of approximately 120 time correlated star positions of five images each, and 600 satellite images when passive satellites are observed.

4.42 Timing Reduction Procedures

A portable crystal clock, synchronized with the crystal oscillator maintained by the U. S. Naval Observatory (A. 1 time), is used to set the master clock at the satellite observing station. Therefore, the station time is referenced in effect to WWV time signals as transmitted (not received), or to UTC. Two VLF receivers at the station and frequencies monitored provide an accurate rate determination of the master clock at the station. The uncertainties in the station timing relative to WWV are estimated to be less than ±150 microseconds for passive satellite observations [Satellite Triangulation in the Coast and Geodetic Survey, 1965, p. 7].

For the GEOS-A Satellite, the APL correction bulletins were used to determine the time the satellite flash was triggered. The U. T. C. epoch of the satellite flash is then converted to preliminary U. T. 1 prior to plate reduction. Preliminary values of the U. T. 1 - U. T. C. correction are taken from the bulletins published weekly by the U. S. Naval Observatory. At a later date, the timing records are corrected by use of the U. S. Naval Observatory "Time Signals Bulletin"(the bulletins are generally published about a year in arrears). The final correction from the "Times Signals Bulletin" is obtained by applying the seasonal variation "S" listed in Section 6 of the Bulletin to the U. T. 2 - U. T. C. values listed in Section 5 of the Bulletin. The preliminary and final U. T. 1 times determined from the bulletin corrections are referred to the old conventional longitude of the U. S. Naval Observatory , Washington [Preuss, 1966, p. 151]. The time record submitted to the Geodetic Satellite Data Center consists of preliminary and final values of this U. T. 1 epoch at which the satellite was triggered to flash. (Note the 44 ms difference between the old and current adopted longitudes of the U. S. N. O.)

Passive satellite timing reductions are essentially the same as the active case. The station camera time is reduced to U. T. C. and values of preliminary and final U. T. 1 (referred to the old longitude of the U. S. N. O.) are listed in the timing record. In the past, all passive observations have been used to determine simultaneous observations for triangulation adjustment at ESSA. In this procedure the results of single camera orientation (plate reduction) are used to determine the directions to the approximately 600 satellite images on a plate. The satellite image directions for two or more plates (from different stations observing the satellite simultaneously) are used to form a preliminary intersection, and to geometrically determine a preliminary position and slant range for each satellite image. The geometrically determined slant range is then used to determine the light time correction for the satellite and other satellite image corrections which will be mentioned in the next section. The light time correction is used to antedate the epoch of the station observation to the satellite. The U. T. 1 times at the satellite are then used to compute a fictitious point by fitting the 600 - 800 satellite images to a fifth order polynomial. From the curves obtained from two or more stations, a fictitious point may be chosen as near the center of both plates as possible to represent a simultaneous observation [Satellite Triangulation in the Coast and Geodetic Survey, 1965, p. 14]. The procedure has been used by ESSA with great success for passive ECHO-1 satellite observations; however, the exact procedure that will be used to provide output data to NASA for passive satellites has not been determined.

4.43 Star Updating Procedures

The ESSA has used the SAO Catalogue for reducing GEOS-A observations. The SAO Catalogue has been subdivided into six units arranged by magnitude and reliability, and stored on magnetic tapes. The first three units contain all stars of magnitude 7.0 or less, and the second three units contain all stars of magnitude 8.0 or less. Each unit contains stars of designated positional accuracy for the epoch 1950.0, and all stars have a positional accuracy of at least $\pm 0''.4$ [Schmid, 1966, p. 9].

The star positions are updated to the apparent place using the following procedure. Proper motion corrections are applied by the use of equations 2.2 and 2.3. The time in tropical years, used in equation 2.3 is the nearest Besselian Year minus the catalogue epoch (1950.0). The correction for precession to the nearest Besselian Year is then applied by use of equations 2.15 and 2.16 where T again refers to the nearest Besselian Year. The stellar coordinates, corrected for proper motion and precession, are then converted to the right ascension-declination system as outlined in Section 2.125.

The stellar coordinates are then corrected to the apparent place at the epoch of observation by use of the "Independent Day Numbers"

$$\begin{aligned} \alpha &= \alpha' + f + g \sin (G + \alpha) \tan \delta' + h \sin (H + \alpha) \sec \delta' + \mu_\alpha \tau + J \tan^2 \delta' \\ \delta &= \delta' + g \cos (G + \alpha') + h \cos (H + \alpha') \sin \delta' + i \cos \delta' + \mu_\delta \tau + J' \tan \delta' \end{aligned} \tag{4.38}$$

where f, g, h, G, H, and i are Independent Day Numbers which are stored on magnetic type for computer use; α', δ' are the uncorrected star coordinates, α, δ are the corrected star coordinates, J and J' the second order Day Numbers, and $\mu_i \tau$ are proper motion corrections.

The apparent star positions are updated to the observed position in the first portion of the single camera orientation program.

4.44 Plate Reduction Procedures

The ESSA employs an extremely rigorous plate measurement and reduction procedure in the single camera orientation program which is similar to the general technique outlined in Section 3.

For plate measurement, diapositives are made from the original photograph negatives. The diapositive plates allow a comparator operator to easily determine the correct positions of white satellite and star images with the black comparator cross hairs. Eight drill holes are then made in the plate. The holes are drilled near the center of the four fiducial marks (at the corners of the plate) and midway between the fiducial marks on the sides of the plate. The drill holes can be measured with greater precision than the fiducial marks, and are used to determine the initial plate coordinate system. The time to measure the 600-800 passive satellite images and 120-150 stars with five images each is three eight-hour shifts. Each plate is measured by one person to avoid differential bias between different measurements. The positions of the eight drill holes are measured three times at the beginning and end of each shift and at two and one-half hour intervals during the shift. If the comparator operator notices the average of the three measurements of the drill hole differs by three microns between the beginning and end of a two and one-half hour interval, the measurements made during the interval are re-measured. Each of the star images (five images for one star trail) and the satellite images are measured in the direct or 0° position and then in the 180° or reverse position [Satellite Triangulation in the Coast and Geodetic Survey, 1965, p. 13].

The measured comparator coordinates are then processed with a computer program which transfers the comparator coordinates to a coordinate system near the center of the plate which is determined by the intersection of the lines connecting the four drill holes at opposite corners of the plate. The computer program also applies comparator calibration corrections similar to the corrections outlined in Section 3.212. The comparator calibration consists of extremely rigorous measurements of a precisely calibrated grid. The results of the rigorous measurements are reduced in a least squares adjustment [Schmid, 1964b, pp. 18-19].

The measured coordinates obtained from each two and one-half hour set of measurements are combined in a "Patching" program to produce two refined sets of image measurements for the images measured in the direct and reverse plate position. The two refined sets of image measurements are then "Matched" by combining the "Patched" star coordinates from the images made in the direct and reverse plate positions to produce a single refined set of image measurements.

After the refined image measurements are available, a "Star Identification" program is used to select reference stars (from the ESSA modified SAO Catalogue tape). The program uses the refined star image measurements, preliminary camera orientation parameters (azimuth, elevation, and roll), and the observation time (in U. T. 1) to determine the reference stars to be used and then to update the stars to the apparent position as outlined in Section 4.43.

After the reference star positions and refined plate measurements have been determined, a "Single Camera Orientation" program is used to determine the final plate orientation parameters. The single camera orientation is similar to the previously described plate reduction procedure outlined in Section 3; however, the ESSA completely calibrates each plate during the plate reduction. The <u>interpreted</u> camera reference system shown in Figure 8 is used to compute the orientation matrix.

The Z axis in the figure is the astronomic zenith of the observer, the X axis is on the direction of the observer's astronomic north, and the Y axis is perpendicular to the XZ plane. The orientation angles of the camera's principal axis are shown in Figure 8, the astronomic azimuth (ψ) positive when measured from North (X) to the East (Y), the astronomic elevation (ω) positive when measured from the XY plane toward the zenith, and the swing of the camera ($\varkappa$) positive when measured from the $\bar{x}$ axis of the plate toward the $\bar{y}$ axis.

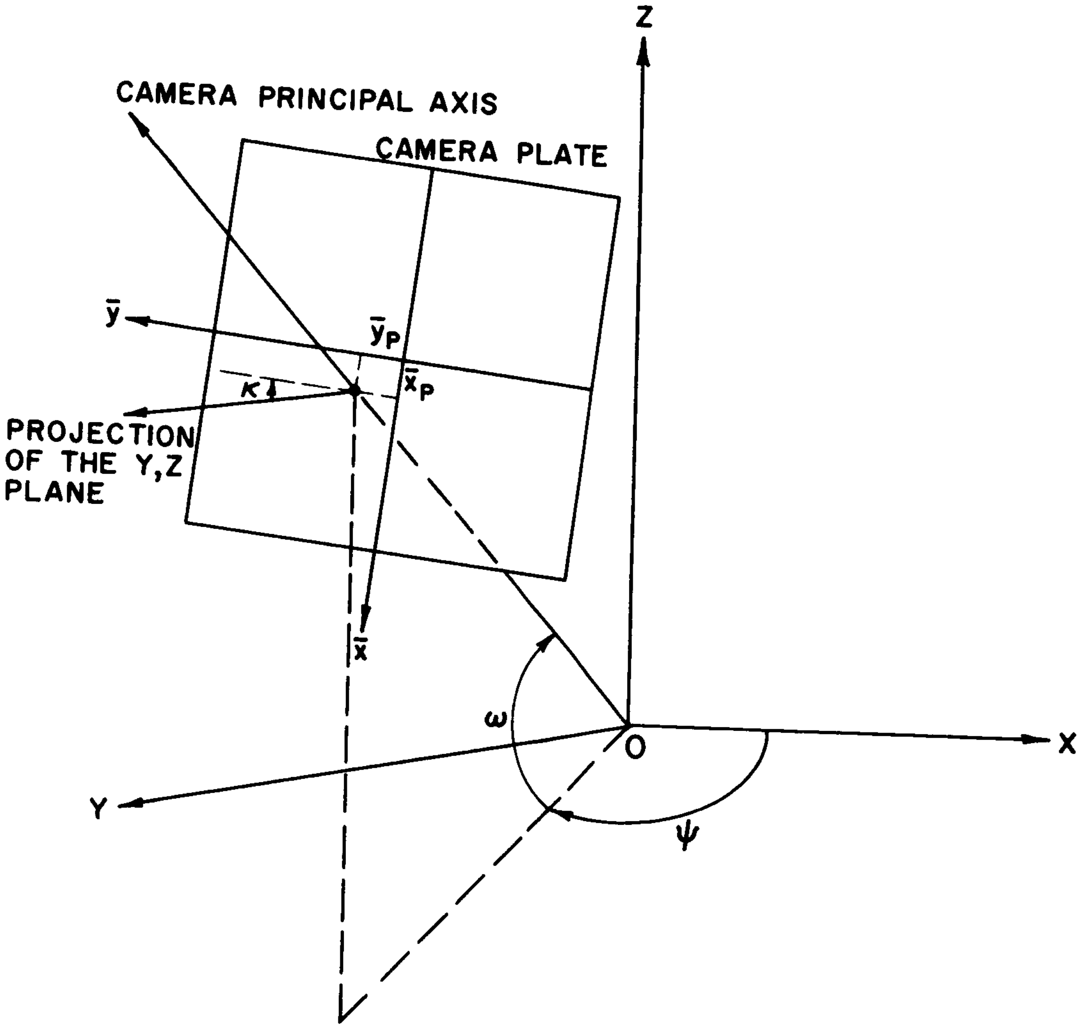
The cartesian coordinates of an arbitrary star image are then computed from

$$M = \frac{\cos\varphi \sin\delta - \sin\varphi \cos\delta \cosh}{\cos Z} ,$$

$$N = \frac{-\cos\delta \sin h}{\cos Z} , \tag{4.39}$$

$$Q = 1 ,$$

FIGURE 8
THE ESSA CAMERA ORIENTATION SYSTEM
(interpreted)



where φ is the geodetic latitude of the observing station, δ is the declination of the star, h is the hour angle of the star, and Z is the observed zenith distance of the star. The M, N, Q coordinates may be derived by computing the azimuth and altitude vector (u, v, w) to the star image and then dividing the X and Y coordinates by the Z coordinate.

The orientation matrix elements (analogous to equation 3.9) used by the ESSA to determine the relationship between the measured plate coordinates ($\bar{x}$, $\bar{y}$, c) referred to the projection center are ,

$$\begin{aligned}
A_1 &= -\cos\psi \cos\varkappa + \sin\psi \sin\omega \sin\varkappa \ , \\
A_2 &= -\cos\omega \sin\varkappa \ , \\
A_3 &= \sin\psi \cos\varkappa + \cos\psi \sin\omega \sin\varkappa \ , \\
B_1 &= -\cos\psi \sin\varkappa - \sin\psi \sin\omega \cos\varkappa \ , \\
B_2 &= \cos\omega \cos\varkappa \ , \\
B_3 &= \sin\psi \sin\varkappa - \cos\psi \sin\omega \cos\varkappa \ , \\
C_1 &= \sin\psi \cos\omega \ , \\
C_2 &= \sin\omega \ , \\
C_3 &= \cos\psi \cos\omega \ .
\end{aligned} \tag{4.40}$$

The projective equations (analogous to equation 3.6) are

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ z \end{pmatrix} = \begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{pmatrix} \begin{pmatrix} M \\ N \\ Q \end{pmatrix} \tag{4.41}$$

The above equations briefly outline the general technique used. A complete description of the solution would be an intricate study of great length. Consequently, the author will simply outline the general way in which corrections are applied and what the output values of the plate reduction are.

The complete single camera orientation program currently includes 20 parameters which can be used in solution of the projective equation. The interpreted parameters are the azimuth, elevation and roll of the camera axis (ψ, ω, $\varkappa$), the coordinates of the principal point ($\bar{x}_p$, $\bar{y}_p$), different principal distances to the $\bar{x}$ axis and $\bar{y}$ axis from the projection center (c_x, c_y), a correction for the nonperpendicularity of the comparator axis (ϵ), the coordinates of the point of origin of radial and tangential distortion ($\bar{x}_s$, $\bar{y}_s$), radial distortion coefficients (K_1, K_2, K_3), tangential distortion coefficients and angle from the $\bar{x}$ axis to the direction of maximum distortion (K_4, K_5, $\hat{\varphi}$), and astronomical refraction coefficients (η_1, η_2, η_3, η_4). The program has the following thirteen options:

1. The principal distances to the $\bar{x}_p$ axis (c_x) can be set equal to the principal distance to the $\bar{y}_p$ axis (c_y) or the two values can be different.

2. Independently determined azimuth-elevation can be introduced as a constraint to the camera orientation.

3. Common weight matrices may be given for stars with known right ascension and declinations or individual weights may be assigned to each star.

4. Common weight matrices may be given for all plate points or individual weights may be assigned to each plate point.

5. Refraction coefficients may be corrected in the program during one additional iteration after a satisfactory maximum solution. They are enforced at all other times.

6. The program can be used to compute a minimum solution using only a set number of designated stars in the orientation. Thereafter the program proceeds to a maximum (all stars) solution or to a "pre-run."

7. A "pre-run" can be programmed to analyze the input data.

8. The program can be used to compute and print out the distortion curves computed during the plate reduction.

9. The final adjusted orientation parameters (ψ, ω, $\varkappa$) can be transformed to the latitude and longitude of a designated station and the orientation parameters can be corrected for polar motion.

10. The orientation parameters can be weighted.

11. The written output data can be punched on cards.

12. The number of iterations can be controlled for either a maximum or minimum (option 6) solution.

13. When using fictitious data a selected mean error of unit weight can be introduced for the computation of all mean errors for the unknowns in the solution.

The options selected for a particular camera orientation are printed out in an easily readable form on the computer output listings.

Basic measurement corrections to the input for each star used in the single camera orientation are given in the following paragraph.

The x_1 measurement of each star image is corrected for nonperpendicularity by the following formula:

$$\overline{x}_i = x_i + \epsilon \overline{y}_i \quad , \tag{4.42}$$

where $\overline{y}_i$ is the y measurement of the image, and ϵ is an adjustable correction for the nonperpendicularity of the comparator axis.

The measured coordinates are corrected for radial and tangential lens distortion by an reiterative technique. The basic formulas used for radial distortion are

$$\begin{aligned} d_i^{\,2} &= (\overline{x}_i - \overline{x}_s)^2 + (\overline{y}_i - \overline{y}_s)^2 \quad , \\ \frac{\Delta R}{d_i} &= K_1\, d_i^{\,2} + K_2\, d_i^{\,4} + K_3\, d_i^{\,6} \quad , \end{aligned} \tag{4.43}$$

where $\overline{x}_i$, $\overline{y}_i$ are defined in equation 4.42; $\overline{x}_s$, $\overline{y}_s$ are adjustable coordinates of the origin of distortion; K_1, K_2, K_3 are adjustable radial distortion coefficients; and $\frac{\Delta R}{d_i}$ is the measure of the radial distortion used later. The basic formulas used for tangential distortion are

$$\begin{aligned} \Delta T &= K_4\, d_i^2 + K_5\, d_i^4 \quad , \\ \Delta TS &= \Delta T \sin \varphi \ , \\ \Delta TC &= \Delta T \cos \varphi \quad , \end{aligned} \tag{4.44}$$

where ΔT is a measure of tangential distortion, φ is the rotation angle from the $\overline{x}$ axis to the axis of maximum tangential distortion, and K_4 and K_5 are tangential distortion coefficients. The correction for radial and tangential distortions are then combined to form the equation

$$\bar{\bar{x}}_i = \bar{x}_i - \frac{\Delta R}{d_i}(\bar{x}_i - \bar{x}_s) - \frac{2(\bar{x}_i - x_s)(\bar{y}_i - \bar{y}_s)}{d_i^{2}} \Delta TS + \left[\frac{2(\bar{x}_i - \bar{x}_s) + d_i^{2}}{d_i^{2}}\right] \Delta TC ,$$

$$\bar{\bar{y}}_i = \bar{y}_i - \frac{\Delta R}{d_i}(\bar{y}_i - \bar{y}_s) - \frac{2(\bar{x}_i - x_s)(\bar{y}_i - \bar{y}_s)}{d_i^{2}} \Delta TC + \left[\frac{2(\bar{y}_i - \bar{y}_s) + d_i^{2}}{d_i^{2}}\right] \Delta TS \cdot \tag{4.45}$$

The solution, as mentioned earlier, is then iterated by setting in the second and following terms, $\bar{x}_i$ and $\bar{y}_i$ equal to the previously computed respective $\bar{\bar{x}}_i$ and $\bar{\bar{y}}_i$ until the change in $\frac{\Delta R}{d_i}$ and ΔT converge to a pre-set tolerance.

The next step applies to stellar (or GEOS-A) image whose right ascension and declination are not known. Since the GEOS-A flashes were reduced in the same manner as unknown star images, the following brief outline of the procedure employed is given. The inverse of the orientation matrix (equation 4.41) is used to compute values of azimuth and elevation of the unknown image. The elevation is then used to compute the astronomic refraction at the image elevation (the type of refraction model will be discussed later). The refraction is then removed from the elevation of the image and a right ascension-declination to the unrefracted image is computed. Finally, the diurnal aberration correction for the image is computed and removed from the right ascension and declination of the image position and the apparent coordinates of the image thus obtained are stored for later use. This entire procedure essentially reduces the unkown image right ascension and declination to the same reference system (i.e., the apparent position) as the "known" stellar images.

All of the stellar coordinates (including the images' coordinates with diurnal aberration removed as outlined in the last paragraph) are corrected for diurnal aberration. The hour angle is computed for each image from the following formula:

$$h_i = LAST_i - \alpha_i \, , \tag{4.46}$$

where h_i is the hour angle of the image, LAST is the local apparent sidereal time of the observation of the image, and α_i is the right ascension of the point. The apparent sidereal time is determined for the first stellar observation of the plate by use of the current Nautical Almanac. The apparent sidereal times for 0^hU. T. of the dates tabulated are interpolated for the epoch of observation in U. T. 1. The value of apparent sidereal time for each subsequent epoch is computed by adding the sidereal interval between the first observation and later observations. The local apparent sidereal time is then computed by subtracting the station's longitude. The diurnal aberration correction is then computed from equation 2. 29 and added to the stellar coordinates.

Refraction corrections are applied to the images by an iterative technique. The formula used is

$$\Delta Z = T^{\frac{1}{2}} W \left(\eta_1 \tan \frac{B}{2} + \eta_2 \tan^3 \frac{B}{2} + \eta_3 \tan^5 \frac{B}{2} + \eta_4 \tan^7 \frac{B}{2} \right) \quad , \tag{4.47}$$

where

$$
\begin{aligned}
W &= \frac{P}{T^2} \ , \\
P &= \frac{P_s}{P_o} \ , \\
T &= \frac{T_s}{T_o} \ , \\
\tan B &= \frac{T^{\frac{1}{2}}}{8.7137} \tan Z_R
\end{aligned}
\qquad (4.48)
$$

In the above formulas, Z_R indicates the refracted zenith distance; T_s denotes the station temperature in degrees Kelvin; P_s denotes the station pressure in millimeters of mercury; P_o, T_o denote the pressure and temperature of the standard atmosphere; and $\eta_1 \ldots, \eta_4$ are refraction coefficients. The refraction coefficients used to reduce GEOS-A observations are

$$
\begin{aligned}
\eta_1 &= 1050.61030 \ , \\
\eta_2 &= 706.11502 \ , \\
\eta_3 &= 262.06086 \ , \\
\eta_4 &= 142.67293 \ .
\end{aligned}
\qquad (4.49)
$$

The refraction equations are iterated until the difference in ΔZ of the n^{th} iteration minus ΔZ of the $(n-1)^{th}$ iteration is within a specified tolerance. The quantity Z_R is replaced in each iteration.

The solution of the projective equations is accomplished by the same general method as outlined in Section 3. Iteration is terminated if one of three conditions is met:

1. A total of four divergent iterations are encountered.
2. A designated tolerance is met.
3. A set number of iterations is met.

After the final adjusted orientation parameters (ψ^a, ω^a, κ^a) are determined, the adjusted parameters can be referred to a mean reference station. If we let $\underset{\approx}{0}$ represent the adjusted orientation matrix from equation 4.41

$$\underset{\approx}{0} = \begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{pmatrix} . \tag{4.50}$$

Then the orientation matrix at the mean reference station is

$$\underset{\approx}{0}^* = \underset{\sim}{R}_3^m \, \underset{\sim}{R}_2^p \, \underset{\sim}{R}_1^o \, \underset{\approx}{0} \quad ,$$

where

$$\underset{\sim}{R}_1^o = \begin{pmatrix} -\cos\lambda \sin\varphi & \sin\lambda & \cos\lambda\cos\varphi \\ -\sin\lambda \sin\varphi & -\cos\lambda & \sin\lambda\cos\varphi \\ \cos\varphi & 0 & \sin\varphi \end{pmatrix} ,$$

$$\underset{\sim}{R}_2^p = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ -x & -y & 1 \end{pmatrix} , \tag{4.52}$$

$$\underset{\sim}{R}_3^m = \begin{pmatrix} -\cos\lambda_m \sin\varphi_m & -\sin\lambda_m \sin\varphi_m & \cos\varphi_m \\ \sin\lambda_m & -\cos\lambda_m & 0 \\ \cos\lambda_m \cos\varphi_m & \sin\lambda_m \cos\varphi_m & \sin\varphi_m \end{pmatrix} ,$$

In the above formulas, φ, λ are the geodetic latitude and longitude of the observing station; φ_m, λ_m are the geodetic latitude and longitude of a designated reference station; and x and y are the polar coordinates at the epoch of observation. The polar coordinates are linearly interpolated from the values given for five day intervals in the "Bulletin for Polar Coordinates" published in Circular D by the Bureau International de l'Heure and corrected to the I. P. M. S. (1900 - 1905) pole.

The orientation parameters referred to the mean station are then used in the passive satellite reduction program. In the satellite direction program, the measured passive satellite coordinates are corrected for nonperpendicularity of the comparator axis, radial distortion, tangential distortion, atmospheric (astronomic-parallactic) refraction, diurnal aberration, satellite phase and parallactic aberration. The satellite phase correction is computed by equation 2.40 or 2.42. The parallactic refraction correction is computed from equation 2.39. The slant ranges used in the parallactic refraction and satellite phase corrections are determined by a "preliminary" geometric intersection of the rays from two cameras which observed the satellite simultaneously. The remaining corrections are computed in the same manner as for the stellar images in the single camera orientation.

The reduction program fits the satellite images to a 5^{th} or 7^{th} degree time series polynomial, then refers the station time of passive satellite observation to the satellite (by selecting a fictitious UT1 time for simultaneity and using light travel time to obtain observation times at each station), and, finally, computes a fictitious intersection between two or more stations in terms of fictitious point plate coordinates and camera orientation parameters, etc. These results are then used by ESSA in satellite triangulation adjustments. The final procedures used for reducing observations of passive satellites (such as Pageos) to be submitted to the National Geodetic Satellite Data Center have not been finalized and must be determined at a later date.

The GEOS-A flash observations as mentioned earlier were reduced as unknown star images (to apparent position). In the single camera orientation program, the preliminary right ascension and declination are determined by reducing the image to a preliminary apparent position and then adjusting the image in the single camera orientation. The adjusted coordinates are then printed out as part of the comprehensive single camera orientation program. These apparent coordinates are, in effect, corrected for parallactic aberration (since the flash time at the satellite is used).

Standard errors of the GEOS-A (or an unknown stellar image) are computed by the following formulas:

$$
\begin{aligned}
m_\alpha &= m\left(\underset{\sim}{Q}^{\circ}{}_{(1,1)}\right)^{\frac{1}{2}} , \\
m_\delta &= m\left(\underset{\sim}{Q}^{\circ}{}_{(2,2)}\right)^{\frac{1}{2}} , \\
m'_\alpha &= \left\{m^{*2}\left[\underset{\sim}{(1)}\right]_{(1,1)} + m^2\underset{\sim}{Q}^{\circ}{}_{(1,1)}\right\}^{\frac{1}{2}} \\
m'_\delta &= \left\{m^{*2}\left[\underset{\sim}{(1)}\right]_{(2,2)} + m^2\underset{\sim}{Q}^{\circ}{}_{(2,2)}\right\}^{\frac{1}{2}}
\end{aligned}
\qquad (4.53)
$$

where m_α, m_δ denote the standard error in right ascension and declination; $\underset{\sim}{Q}^{\circ}$ is the covariance matrix obtained from the plate adjustment (equation 4.54);

(1, 1) and (2, 2) subscripts refer to elements of the $\underset{\sim}{Q}$ matrix; and m is the standard error of an observation of unit weight determined during the plate adjustment; the superscript T refers to the "total" standard errors; m* is the mean error of measurements of the GEOS-A images; and the matrix $[\underset{\sim}{(1)}]$ is determined by equation (4.57).

The covariance matrix used to determine the standard error of an image position due to errors in determining the orientation parameters ($\underset{\sim}{Q}^{\circ}$) is computed from

$$\underset{\sim}{Q}^{\circ} = [\underset{\sim}{(1)}]\ [\underset{\sim}{(2)}]\ \underset{\sim}{N}^{-1}\ [\underset{\sim}{(2)}]^{T}\ [\underset{\sim}{(1)}] \ , \tag{4.54}$$

where

$$\underset{\sim}{N} = \sum_{\underset{\sim}{i=1}}^{n} \underset{\sim}{B}_{oi}^{T}\ \underset{\sim}{W}_{oi}\ \underset{\sim}{B}_{oi} - [(\underset{\sim}{B}_{o}^{T}\ \underset{\sim}{W}_{oi}\ \underset{\sim}{B}_{x})(\underset{\sim}{B}_{x}^{T}\ \underset{\sim}{W}_{oi}\ \underset{\sim}{B}_{x} + \underset{\sim}{W}_{\alpha\delta})_{j}^{-1}\ \underset{\sim}{B}_{x}^{T}\ \underset{\sim}{W}_{oi}\ \underset{\sim}{B}_{o}]_{i} + W_{o} \tag{4.55}$$

The $\underset{\sim}{B}_{o}$ matrix is the matrix of partial derivatives of the entire projective equation with respect to each of the parameters, and the $\underset{\sim}{W}_{pi}$, $\underset{\sim}{W}_{\alpha\delta}$, and $\underset{\sim}{W}_{o}$ matrices are the weight matrices of Plate Observation, of Star and of Orientation Parameters, respectively. The first summation is for all observations. The matrices in the bracket are summed for the n observations of the star. The last term is added once.

The $[\underset{\sim}{(2)}]$ matrix is computed from

$$[\underset{\sim}{(2)}] = \sum_{\underset{\sim}{i=1}}^{n} \underset{\sim}{B}_{pi}\ \underset{\sim}{W}_{pi}\ \underset{\sim}{B}_{o} \ , \tag{4.56}$$

where $\underset{\sim}{B}_{P1}$ represents the matrix of partial derivatives or the measured parameters with respect to the unknown star or satellite position. The $\underset{\sim}{W}_{P1}$ matrix is the weight matrix of the measured parameters and the product summed for n image measurements. Finally, the $[\underset{\sim}{(1)}]$ matrix is determined by

$$[\underset{\sim}{(1)}] = -\left[\sum_{i=1}^{n} \underset{\sim}{B}^{T}_{P1} \quad \underset{\sim}{W}_{P1} \quad \underset{\sim}{B}_{P1} + \underset{\sim}{W}_{\alpha\delta} \right]^{-1} . \tag{4.57}$$

The $\underset{\sim}{W}_{\alpha\delta}$ matrix represents the weight matrix of a weighted star of satellite position.

The standard error of an observation of unit weight (m) of the entire adjustment is computed from

$$m = \left(\frac{\Sigma \underset{\sim}{V}^{T} \underset{\sim}{W} \underset{\sim}{V}}{f} \right)^{\frac{1}{2}} . \tag{4.60}$$

Very briefly, the $\underset{\sim}{V}^{T} \underset{\sim}{WV}$ term represents a summation of three other matrix summations of the form

$$\Sigma \underset{\sim}{V}^{T} \underset{\sim}{W} \underset{\sim}{V} = \Sigma \underset{\sim}{v}^{T} \underset{\sim}{w} \underset{\sim}{v} + \ldots \quad , \tag{4.61}$$

where the $\underset{\sim}{v}$ matrices are residuals of adjusted parameters for the entire plate adjustment, and $\underset{\sim}{w}$ is the weight matrix for the parameter. The three summations represent the residuals and associated weights for the weighted measured parameters, the weighted star observations, and the weighted orientation parameters. The degrees of freedom (f) are determined from the number of observed parameters of finite weight minus the number of unknown parameters.

The method that will be used to compute the standard errors for passive satellite observations submitted to the Geodetic Satellite Data Center is currently being formulated by ESSA, but the final method has not been determined.

4.45 The Final Results

The final results of the plate reduction are forwarded to the Geodetic Satellite Data Center in the format outlined in Appendix 1.

The GEOS-A satellite's coordinates are in the right ascension-declination system and represent the satellite's "apparent topocentric" position (observed position corrected for astronomic refraction and diurnal aberration; see p. 120) at the epoch of the observation. This position differs from the true topocentric position by corrections for diurnal aberration and parallactic refraction.

The epoch of GEOS-A observation refers to the time the flash mechanism was triggered to flash preliminary UT1; consequently, no correction is necessary for parallactic aberration.

The standard errors of the GEOS-A position are the "total" errors in Section 4.44.

It should be noted again that all times given with the final results refer to the old adopted longitude of the U. S. N. O., thus they are 44ms too large.

Procedures for reducing passive satellite data for the Geodetic Satellite Data Center are presently being formulated; however, final decisions involving the exact nature of the data output have not been made.

4.5 THE AERONAUTICAL CHART AND INFORMATION CENTER PROCEDURES

4.51 General

The Aeronautical Chart and Information Center (ACIC) performs the data reduction for plates from PC-1000 Cameras and BC-4 Cameras operated by the U. S. Air Force. The PC-1000 Camera has a 1000 mm focal length, a 200 mm aperture, and uses standard photographic plates [Moss et al., 1966, p. 12]. The BC-4 Camera systems were briefly described in Section 4.41. The camera systems are operated in the fixed mode.

The PC-1000 has a "between the lens" internal shutter and recently has been also equipped with an external capping shutter [Lawrence, 1963, p. 11].

During an active satellite observation, the internal camera shutter system is activated before and after the satellite observation to produce chopped star trails. The shutter is programmed to open for exposures of 2, 1, 0.5, 0.3, and 0.1 seconds, and the shutter is closed 20 seconds between exposures. Two exposure sequences are performed before and after satellite observation, and the shutter remains closed 40 seconds between the two sequences. The shutter is opened for periods ranging from two to two and one-half minutes to record satellite flashes.

4.52 Timing Reduction Procedures

The camera systems are equipped with an electronic console which contains a crystal station clock and a radio receiver. The receiver is used to determine the clock offset and rate from the time signal received.

The timing signals received at the observing station are recorded on one channel of magnetic tape (or in later model systems recorded on one channel of a paper tape), and the clock is recorded on a second channel. If magnetic tape is used, the magnetically recorded signals are reproduced by using a Visicorder (model no. 9066). The record consists of the signal on the edges of the paper, and the station clock signal on the center of the record. The rate and offset of the station clock relative to the radio time signals received are determined by correlating the recorded signals.

During the plate exposure, the shutter action is recorded on the channel previously used for the time signal record. The epoch of shutter action can then be correlated with the clock signals in the same manner as outlined for the radio time signals.

Basically three corrections are applied to the epoch of shutter action. The first correction is a correction for the emission delay within the electronic receiving units. The second correction is a shutter correction for the delayed action of the shutter. Finally, a propagation delay correction is computed from

$$\Delta T = \left(\frac{\psi_e R}{V}\right) \left(\frac{2\pi}{360}\right) , \qquad (4.62)$$

where ΔT is the propagation delay, ψ_e is the angle subtended at the Earth's center between the time signal station and the observing station, R is the mean radius of the Earth, and V is the propagation velocity of the radio signal.

For GEOS-A observations, the time of flash published by the APL was corrected by the current APL Bulletin. The epoch of GEOS-A observations thus refers to the time the satellite flash was triggered in U. T. C.

4.53 Star Updating Procedures

The ACIC used the Boss General Catalogue for determining the coordinates of reference stars that were used in plate reduction. Plans are in progress to use the SAO Catalogue instead of the Boss Catalogue sometime in 1967. The Boss General Catalogue is updated to the mean place at the beginning of the nearest Besselian Year and stored on magnetic tape.

The updating is accomplished by using the method of annual and secular variations. The formulas used are [Harp, 1966, p. 5],

$$\begin{aligned} \alpha_y &= \alpha_m + \text{A.V.}_{\alpha}(T-1950) + \frac{1}{200}\,\text{S.V.}_{\alpha}(T-1950)^2 \,, \\ \delta_y &= \delta_m + \text{A.V.}_{\delta}(T-1950) + \frac{1}{200}\,\text{S.V.}_{\delta}(T-1950)^2 \,, \end{aligned} \tag{4.63}$$

where the A.V._i, S.V._i terms are the catalogued values of annual and secular variation in right ascension and declination; α_y, δ_y are the star's mean right ascension and declination at the beginning of the nearest Besselian Year; α_m, δ_m are the catalogued right ascension and declination; and T is the nearest Besselian Year. Corrections to the star positions for third term variations are not included in the computation.

The coordinates are then updated to the apparent place at the time of observation by means of the Besselian Day Numbers,

$$\begin{aligned} \alpha_A &= \alpha_y + \tau\mu_\alpha + Aa + Bb + Cc + Dd + E + J\tan^2\delta \,, \\ \delta_A &= \delta_y + \tau\mu_\delta + Aa' + Bb' + Cc' + Dd' + J'\tan\delta \,, \end{aligned} \tag{4.64}$$

where α_A, δ_A are the star's apparent coordinates; μ_α, μ_δ are the catalogued proper motions; τ is the fraction of a Besselian Year from the nearest beginning of the Besselian Year; A, B, C, D, and E are Besselian Day Numbers; a, b, c, . . . , a', d' are star constants, and J and J' are second order Day Numbers. The fraction of the year , Besselian Day Numbers and second order Day Numbers are obtained from the current Nautical Almanac and linearly interpolated to the U. T. C. epoch of observation. The star constants are computed from

$$
\begin{aligned}
a &= m/n + \sin\alpha_y \tan\delta_y & a' &= \cos\alpha_y \\
b &= \cos\alpha_y \tan\delta_y & b' &= -\sin\alpha_y \\
c &= \cos\alpha_y \sec\delta_y & c' &= \tan\epsilon_y \cos\delta_y \\
 & & &\quad -\sin\alpha_y \sin\delta_y \\
d &= \sin\alpha_y \sec\delta_y & d' &= \cos\alpha_y \sin\delta_y
\end{aligned}
\qquad (4.65)
$$

$\epsilon_y =$ The mean obliquity of the ecliptic and is determined from the Nautical Almanac for the nearest Besselian Year (T).

$$m/n = 2.29887 + 0.00237\,(T-1900)$$

Diurnal aberration corrections are then applied by use of equations 2.29. Refraction corrections are also applied in the first part of the plate reduction procedure to obtain the observed star's position.

4.54 Plate Reduction Procedures

The ACIC employs the plate reduction technique developed by Brown [Brown, 1964]. The camera reference system shown in Figure 3 is used in the plate reduction.

A complete camera calibration is performed for each camera to determine the lens distortion coefficients for subsequent plate reductions. Recalibration is accomplished when the camera station is moved, and also at any time when there is apparently something wrong with the data being acquired by the camera, or if the crew should feel that something might have happened to the camera.

Approximately 25 or 30 stars equally distributed and within one inch of the satellite path are chosen for reference points. The image in the chopped star trail which is most nearly the same size as the satellite image is selected and numbered for subsequent measurement. The stars chosen are identified by matching the plate images with the ACIC Stellar Atlas.

The Stellar Atlas is a chart of Boss Catalogue star positions and is the same "scale" as the photographic plate.

The plate is then measured on a Mann semi-automatic comparator. The semi-automatic comparator features a completely enclosed measuring engine which maintains a constant measuring environment. The measuring engine has an air-bearing guide system which greatly reduces undesirable comparator effects. The comparator projects the plate images onto a large screen with a "bull's eye" mark in the center of the screen. The comparator operator uses a lever to automatically position the star image to be measured in the "bull's eye." The operator then activates an "automatic mode" control and the comparator automatically finds the most dense point of the image and punches the $\overline{x}$, $\overline{y}$ coordinates of the point on cards for further use. The semi-automatic comparator is calibrated daily.

Radial and tangential lens distortions and lens decentering are applied to the measured coordinates by equations 3.24 and 3.25.

Refraction corrections are then applied to the stellar coordinates. The camera elevation is used to determine the zenith distance and the Garfinkel refraction formula (equation 2.33) is used to determine the astronomic refraction by an iterative technique. The approximate refraction coefficients are determined by using Table 5. The table is an abbreviated version of the Garfinkel refraction table [Garfinkel, 1944].

The argument for the table is $\overline{\theta}$ which is computed by reiteration of equation 2.34. The approximate refraction coefficients obtained from the table are used as constrained parameters in the plate adjustment.

TABLE 5

GARFINKEL REFRACTION COEFFICIENTS USED BY ACIC

$\bar{\theta}$	η_1	η_2	η_3	η_4	η_5
10°	87.1	0	0	0	0
20°	178.3	0.2	0	0	0
30°	278.5	0.8	0	0	0
40°	394.2	2.3	0	0	0
50°	535.4	5.5	0.1	0	0
60°	718.6	12.4	0.4	0	0
65°	834.5	18.6	0.7	0	0
70°	974.0	27.9	1.4	0.1	0
75°	1145.3	42.4	2.7	0.2	0
80°	1361.2	65.5	5.1	0.5	0
85°	1640.4	103.4	10.4	1.2	0.1
89°	1928.5	152.2	18.7	2.6	0.4

A preliminary orientation using the stellar images is then computed by using only five identifiable stars to compute better approximate values of the orientation parameters, and to edit all of the input stellar and measurement parameters. The editing prevents "blunders" from entering the final solution.

The ACIC uses mean sidereal time (θ_m) in the plate reduction. The mean sidereal time is determined from

$$\theta_m = \theta_{ym} + [N + UT](1.0027379093) , \tag{4.66}$$

where, θ_{ym} is the mean sidereal time at 0^hU. T. at the beginning of the

Besselian Year, N is the number of days from the beginning of the Besselian Year, and UT is the fraction of a day from 0^hUT to the epoch of observation.

The projective equations for each star image are then solved as outlined in Section 3.23. The adjustable parameters are ψ, ω, κ, x_p, y_p, c, η_1, η_2, η_3, and η_4. The parameters are defined in Section 3.23.

After the adjusted parameters are obtained from the plate solution, the satellite image coordinates (corrected for lens distortion) are used to determine the satellite's azimuth and elevation by use of equation 3.10.

The refracted satellite elevation obtained from the plate reduction is then corrected for refraction using the Garfinkel model and the adjusted coefficients from the plate adjustment. The unrefracted elevation and azimuth are then converted to the right ascension and declination system by use of formulas similar to those given in equation 4.22.

Estimates of the standard error of the satellite image in right ascension and declination are also given; however, the author did not determine how the standard errors were computed.

4.55 Final Results

The final results of the plate reduction are forwarded to the Geodetic Satellite Data Center in the format outlined in Appendix 1.

The satellite's coordinates are in the right ascension-declination system, and represent the satellite image position referred to the geometric (i.e., corrected for astronomic refraction) topocentric position at the epoch of observation. Parallactic refraction corrections are not applied to the satellite position.

The epoch of observation of the GEOS-A satellite refers to the epoch the flash mechanism was triggered in U. T. C.; consequently, no correction is necessary for parallactic aberration.

5 CONCLUSIONS AND RECOMMENDATIONS

The satellite data processing methods used by the various agencies have been outlined in Section 4. In conclusion, the author will recommend procedures and corrections to be applied to the data from the agencies for further geodetic investigation. As mentioned in Section 4. 1, the procedures must be regarded as preliminary since they have not been reviewed by the agencies. Consequently, the recommended corrections must also be regarded as preliminary.

A summary of the corrections applied by the agencies to the satellite or star positions as outlined in Section 4 is given in Table 6.

The conclusions and recommendations are developed in the remaining three sections of the report. First, general consideration relating to the preprocessing procedure are reviewed. Second, the specific preprocessing procedures for each agnecy are outlined. Finally, recommendations are offered concerning future data preprocessing.

TABLE 6

AGENCIES' DATA CORRECTIONS

	AGENCY					
	NASA (MOTS)*			SAO		
	Star Position	Satellite Position	Time	Star Position	Satellite Position	Time
Precession	AE					
Proper Motion	AE			AE		
Nutation	AE					
Annual Aberration	AE				AE	
Diurnal Aberration	AE					
Astronomic Refraction	AE	RE		AA	AA	
Parallactic Refraction		AE				
Satellite Aberration (light time)			AA^{G}			AE^{1}
Satellite Phase (passive satellites)						
Time: UTC to UT1						
Time: UTC to AS						AE
Plate Reduction	Photogrammetric			Astrometric		
Catalogue	SAO			SAO		

* indicates the agency only observes active satellites for geodetic purposes.

AE indicates the correction is applied explicitly.

RE indicates the correction is removed explicitly.

AA indicates the correction is automatically made in the agency's processing procedures.

AE^{1} indicates the epoch of satellite observation is referred to the station time for GEOS-A observations.

MP indicates the agency uses the 1950.0 star positions.

UP indicates the corrections are applied for passive satellites only.

G (as a superscript) indicates the correction is applied to GEOS-A observations.

TABLE 6 (Continued)

AGENCIES' DATA CORRECTIONS

	AGENCY					
	ESSA (CGS)			ACIC*		
	Star Position	Satellite Position	Time	Star Position	Satellite Position	Time
Precession	AE			AE		
Proper Motion	AE			AE		
Nutation	AE			AE		
Annual Aberration	AE			AE		
Diurnal Aberration	AE	RE		AE		
Astronomic Refraction	AE	RE		AE	RE	
Parallactic Refraction		UP				
Satellite Aberration (light time)		UP	AA^G			AA^G
Satellite Phase (passive satellites)		UP				
Time: UTC to UT1		AE**				
Time: UTC to AS						
Plate Reduction	Photogrammetric			Photogrammetric		
Catalogue	SAO			Boss (being changed to SAO)		

* indicates the agency only observes active satellites for geodetic purposes.

AE indicates the correction is applied explicitly.

RE indicates the correction is removed explicitly.

AA indicates the correction is automatically made in the agency's processing procedures.

AE^1 indicates the epoch of satellite observation is referred to the station time for GEOS-A observations.

MP indicates the agency uses the 1950.0 star positions.

UP indicates the corrections are applied for passive satellites.

G (as a superscript) indicates the correction is applied to GEOS-A observations.

** referred to the old adopted longitude of the U. S. N. O. (times given are 44 ms too large).

5.1 GENERAL PREPROCESSING CONSIDERATIONS

As outlined in Section 2, the satellite position to be used for further geodetic investigation is the true topocentric position referred to the true equator and equinox of the epoch of the observation.

The epoch of satellite observation should be in the U.T.1 system and referred to the time the light left the satellite. The satellite observing stations, however, use U.T.C. to determine the observation epoch, and then, in some instances, change the U.T.C. epoch to another time system. The difference between U.T.1 and U.T.C. is available in preliminary and final bulletins from the U.S. Naval Observatory [Preuss, 1966, pp. 150-162]. The final U.T.1 - U.T.C. corrections, however, are not immediately available; therefore, the author has referred the time of satellite observation to U.T.C.

The U.T.C. epoch, of course, depends on the source of time transmission. For the GEOS-A satellite, the U.T.C. time of flash is referred to the WWV transmissions (e.g., to NBS(UA)). Agencies (i.e., SAO or ESSA) using the portable clock method of station time synchronization also use the same time reference (NBS(UA) or A.1). Since only ESSA and SAO observe passive satellites (for geodetic purposes) the precise nature of U.T.C. used by other agencies is immaterial. However, if in the future the data from other agencies is used the source (i.e., the frequency standard regulating the U.T.C.) must be determined. Unfortunately, the data from the GSDC does not indicate the time signal source.

In order to determine the satellite image corrections (parallactic refraction, satellite aberration, and satellite phase) the range to the satellite must be known. A computer program is currently available in the Geodetic Science Computer Library which will compute the satellite's range. The library number of the program is S 50, and the program will compute the range using the satellite orbital elements, the station position, and the epoch of observation.

The parallactic refraction correction requires that the pressure and temperature at the observing station is available. This information is not presently available from the GSDC format. The author will discuss this problem further in Section 5. 3.

All of the agencies currently submit satellite observations to GSDC in the format given in Appendix 1.

A summary of the corrections to be applied are given in Table 7. The corrections are based on the agency procedures outlined in Section 4. The "annual aberration" correction listed in the table refers to a specific agency and will be explained in Section 5. 1; the remaining corrections are explained in Section 2.

The correction for diurnal aberration implies that the observing stations' latitude and longitude are known. Since the satellite observation format does not contain the station coordinates, the coordinates must be determined from a current station position listing. The datum used is not critical for this purpose.

TABLE 7

PREPROCESSING CORRECTIONS

CORRECTION	AGENCY			
	NASA (MOTS)	SAO	ESSA	ACIC
Annual Aberration	No	Yes	No	No
Precession and Nutation	No	Yes	No	No
Diurnal Aberration	No	Yes	Yes**	No
Time: UT1 - UTC	No	No	Yes***	No
Time: AS - UTC	No	Yes	No	No
Parallactic Refraction	No**	Yes	Yes**	Yes**
Satellite Aberration (Light Time)	No**	Yes	No	No**
Satellite Phase (Passive Satellites)	N. Obs.	Yes*	No	N. Obs.

* indicates the correction is for passive satellite observations only.

** indicates the correction is for GEOS-A observations only.

*** indicates that 44 ms must be subtracted from given UT1 to get correct UT1.

N. Obs. indicates the agency does not observe passive satellite for geodetic purposes.

To compute the passive satellite phase correction, the radius of the satellite must be known and the type of satellite (i. e., highly reflective or diffusive) must be determined.

Shimmer effects have been ignored in this section since, as outlined in Section 2. 23, the effects cannot be corrected by an explicit mathematical expression.

5.2 SPECIFIC PREPROCESSING PROCEDURES

5.21 Minitrack Optical Tracking System Preprocessing Procedures

As outlined in Section 4.2 and illustrated in Table 7, the NASA/MOTS data submitted to the GSDC consists of the satellite's true topocentric right ascension and declination referred to the true equator and equinox at the epoch of observation. The epoch of observation is the U.T.C. time the GEOS-A flash was triggered. Consequently, no preprocessing procedures are necessary.

Two procedures currently used to process the data are of interest. The first point is the approximation used to determine the mean obliquity of the ecliptic and the precessional elements as shown in equations 4.4 and 4.5. The author has determined that the approximations are valid.

The second point of interest is the manner in which the aberration corrections are computed. The complicated aberration correction given in equation 4.8 can be interpreted as the "Conventional Besselian Day Number" method if we assume that the C and D Besselian Day Numbers are represented by

$$\begin{aligned} C \simeq C_L &= -K \cos \epsilon_m \cos \odot , \\ D \simeq D_L &= -K \sin \odot , \end{aligned} \tag{5.1}$$

where K is the aberration constant ($20''.47$), ϵ_m is the mean obliquity of the ecliptic, and $\odot$ is the apparent longitude of the Sun. The author computed the "provisional" Day Numbers C_L and D_L and compared them with the values tabulated in the Nautical Almanac for six different months in 1966. The values of $\odot$ and ϵ_m were obtained by the procedures outlined in Section 4.23 for 0^hE. T. on the first day of each month. The results are given in Table 8. The value of the tabulated Nautical Almanac C and D values are given and the difference between the tabulated values and computed values are listed in seconds of arc. The maximum difference is less than 0".02 throughout the year and the error introduced by the procedure can be considered to be negligible.

TABLE 8

DIFFERENCES BETWEEN THE BESSELIAN DAY NUMBERS C AND D AND PROVISIONAL VALUES C_L AND D_L (seconds of arc)

DATE	C	$C - C_L$	D	$D - D_L$
1/1/66	-3".311	-0".099	12".139	-0".011
1/3/66	-17.645	-0.003	6.998	-0.019
1/5/66	-14.342	+0.008	-13.218	-0.013
1/7/66*	2.848	+0.008	-20.235	-0.005
1/7/66**	2.854	+0.014	-20.234	+0.005
1/9/66	17.421	-0.005	- 7.633	-0.003
1/11/66	14.772	-0.004	12.621	-0.012

* C and D are referred to the mean equinox of 1966

** C and D are referred to the mean equinox of 1967

5.22 Smithsonian Astrophysical Observatory Preprocessing Procedures

The data submitted to GDSC by the SAO consists of the satellite image right ascension and declination referred to the mean equator and equinox of 1950.0 and the epoch of observation. The time of observation is in the A.S. system and represents the time the light is received at the station. For GEOS-A observations, the epoch also refers to the time of maximum light intensity. The corrections to be applied are summarized in Table 7; however, the correction for annual aberration requires further discussion.

The SAO corrects the satellite image coordinates for annual aberration as shown in equation 4.34. The provisional Day Numbers C_s and D_s should correspond to C_o and D_o as shown in Section 2.122, equation 2.14. The author computed the values of C_o and D_o for 0^h E.T. of the first day of each month. The computations were made by using equation 2.11. The values of C and D were taken from the current Nautical Almanac, and the values of C_s and D_s were computed from equations 4.35 and 4.36. The values of C_o and D_o, and the differences of $C_o - C_s$ and $D_o - D_s$ are tabulated in Table 9. The differences can reach 0".08 and should be corrected.

A flow chart showing the correction procedures is given in Figure 9, and descriptions are given below. The numbers correspond to the numbered blocks on the flow chart.

1. First the Julian Date at 0^hU.T. must be computed. The Julian Date could be ascertained from the current Nautical Almanac; however, a computer program subroutine is currently available in the Geodetic Science Department of The Ohio State University that will compute the Julian Date for a given day, month, and year.

TABLE 9

DIFFERENCES BETWEEN THE BESSELIAN DAY NUMBERS C_o AND D_o AND PROVISIONAL VALUES C_s AND D_s
(in seconds of arc)

DATE	C_o	$C_o - C_s$	D_o	$D_o - D_s$
1/1/66	-3.239	+0.066	20.153	+0".003
1/3/66	-17.620	+0.024	7.073	+0.063
1/5/66	-14.389	-0.043	-13.157	+0.052
1/7/66	- 2.776	-0.073	-20.247	-0.014
1/9/66	17.391	-0.035	- 7.712	-0.082
1/11/66	14.820	+0.046	12.557	-0.084

2. The Modified Julian Date should then be computed by adding the epoch of observation (converted to a fraction of a day) to the Julian Date at zero hours and subtracting 2400000.5 days.

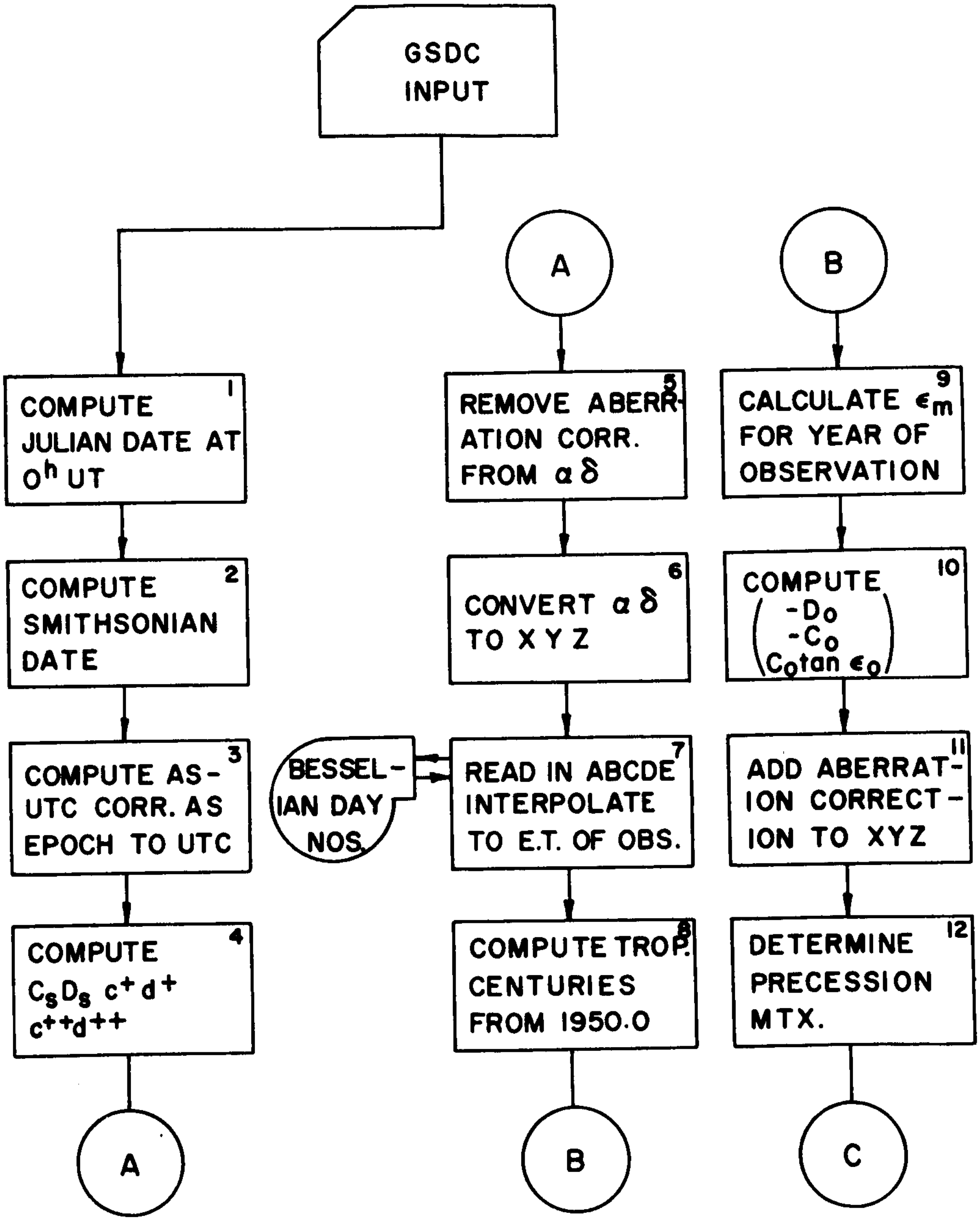
3. The correction A. S. - U. T. C. should then be accomplished by using the values given in Table 3 and the value of t obtained in step 2. The U. T. C. epoch of observation may then be correction by subtracting the A. S. - U. T. C. correction from the A. S. epoch of observation.

epoch of observation.

4. The provisional aberration Day Numbers C_s and D_s may then be computed as shown in equation 4.36 and 4.35 using the Smithsonian Date found in step 2.

FIGURE 9

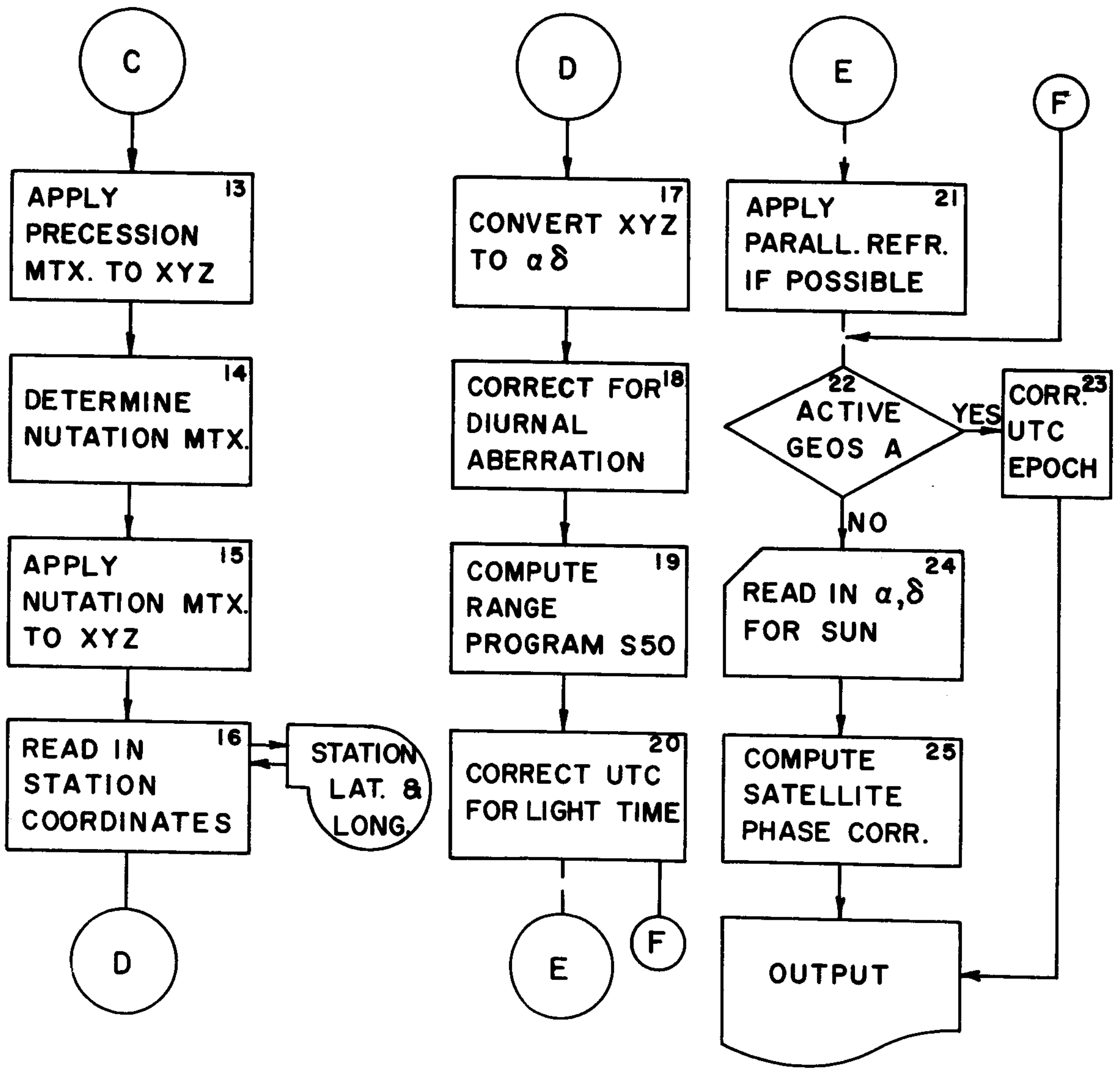
SAO PREPROCESSING FLOW CHART



CONTINUED ON NEXT PAGE

FIGURE 9

SAO PREPROCESSING FLOW CHART
(Continued)



5. The provisional aberration correction can be removed by

$$\begin{aligned} \alpha_s &= \alpha_s^a - C_s c^* - D_s d^* \quad , \\ \delta_s &= \delta_s^a - C_s c^{**} - D_s d^{**} \quad . \end{aligned} \qquad (5.2)$$

The terms C_s and D_s are determined in step 4, and c^*, c^{**}, d^* and d^{**} can be found from equations 4.35 and 4.36. The procedure outlined above involves using the "aberrated" coordinates to find the coordinates not effected by aberration, and, technically, a reiterative solution should be used. The maximum value of the aberrational change will be less than $20\overset{''}{.}47$ (the aberration constant). Therefore, the solution does not need to be reiterated since the trigometric functions will change by less than 10^{-4}.

6. For the next few steps, the cartesian coordinates are needed. The right ascension and declination should be converted to cartesian coordinates by use of equation 2.28.

7. The Besselian Day Numbers A, B, C, D, and E may then be read for the days near the date of observation. The author recommends that the Day Numbers be obtained from magnetic tapes which can be ordered from the U. S. Naval Observatory. The values of A, B, C, and D for 0^h E. T. should be interpolated to the epoch of observation using provisional ephemeris time. The provisional ephemeris time can be determined by adding 36 seconds to the U. T. C. epoch (for 1966.5) [A. E. N. A., 1966, p. vii].

8. The number of tropical centuries from 1950.0 to the epoch of observation must then be determined. The procedure is described in Section 2.121 and equation 2.4.

9. The mean obliquity of the ecliptic for the nearest Besselian Year can be determined from equation 2.12.

10. The values of C_o and D_o can then be determined by use of equation 2.11, the values of C and D determined in step 7, and the mean obliquity of the ecliptic from step 9. The rotation angles in equation 2.11 should be obtained from equation 2.15.

11. The aberration correction referred to the 1950.0 equinox must then be added to the cartesian coordinate vector found in step 6 (as shown in equation 2.13).

12. The precession matrix must be determined from equation 2.15 using T_o = 1950.0 and T as determined in step 8.

13. The precession matrix may then be applied as shown in equation 2.19.

14. The nutation matrix must be determined from equation 2.22. The values used in the matrix can be found from equations 2.24 and 2.25. The values of m and n used in equation 2.24 are functions of the number of tropical years since 1950.0 determined in step 8, and the Besselian Day Numbers A, B and E determined in step 7.

15. The nutation matrix must then be applied by use of equation 2.26.

16. For the remaining corrections, the station latitude and longitude must be known. The station coordinates may either be "stored" on magnetic tape and "read in" from the tape, or "read in" from a punched card.

17. The remaining steps also require that the satellite coordinates be expressed in right ascension and declination. The cartesian coordinate vector obtained in step 16 may be converted to the right ascension-declination system by equations 2.27 and 2.28. The cartesian coordinates, however, should be "stored" for future use.

18. The satellite image position must be corrected for diurnal aberration. The correction is given in equation 2.29. To use the correction the apparent sidereal time at zero hours U.T. should be known (equation 2.32). The mean sidereal time at zero hours U.T. may be obtained from [Explanatory Supplement, 1961, p. 75]:

$$\theta_m(0^hU.T.) = 6^h38^m45\overset{s}{.}836 + 8640184\overset{s}{.}542\ T + 0\overset{s}{.}0929\ T^2\ . \tag{5.3}$$

The T used in equation 5.2 should be determined from the Julian Date at zero hours determined in step 1 minus 2415020.313 which is the Julian Date at 12^hU.T. on 1900 January 0 divided by 36525 (the number of days in a Julian Century). The mean sidereal time (θ_m) may then be corrected to the apparent sidereal time (AST) by use of the $\Delta\psi\cos\epsilon$ term computed in step 14 (by equation 2.25). The formula for the apparent sidereal time at the epoch of observation is

$$AST = \theta_m\,(0^hUT) + UT\ (\ 1.0027379)\ \ \Delta\psi\cos\epsilon\ \ . \tag{5.4}$$

U.T. in the above equation denotes the U.T.C. interval from 0^hU.T. to the epoch of observation and all of the terms are expressed as fractions of a day.

19. The range to the satellite must then be computed from computer program S 50 (see Section 5. 1) using the station coordinates from step 16, the date of observation from the GSDC input card, and the U. T. C. epoch of observation determined in step 3.

20. A light time correction should then be made to reduce the U. T. C. epoch of satellite observation to the time the light left the satellite by use of equation 2. 37.

21. The next correction that should be made is to correct the satellite coordinates for parallactic refraction. As mentioned in Section 5. 1, the correction requires that the temperature and barometric pressure can be determined. The station temperature and pressure are not given in the GSDC format, but if they can be determined from another source, they should be read in and the correction applied by using equation 2. 39. Equation 2. 39, of course, requires the zenith distance of the satellite image, which, in turn, requires another computation. However, since the correction cannot be made with the GSDC data, the author will not outline the procedure.

22. A test is then made to see if the satellite observation was active or passive. The GSDC format column eight will contain a zero if the observation was active, or a one if the observation was passive.

23. If the satellite observation was active (for GEOS-A) the U. T. C. epoch of observation must be antidated by 0. 75 milliseconds to reduce the time to the epoch the satellite flash was triggered. This step terminates the processing of a GEOS-A satellite observation.

24. If the observation was passive, a phase correction should be made to the satellite's coordinates. The phase correction formulas are outlined in equations 2.40 and 2.42. The persons using the reduced data must determine if the satellite is considered to be reflective (equation 2.40) or diffusive (equation 2.42). The right ascension and declination of the Sun for the epoch of observation must be interpolated from the current Nautical Almanac for use in the equations.

5.23 The Environmental Science Services Adminstration Preprocessing Procedures

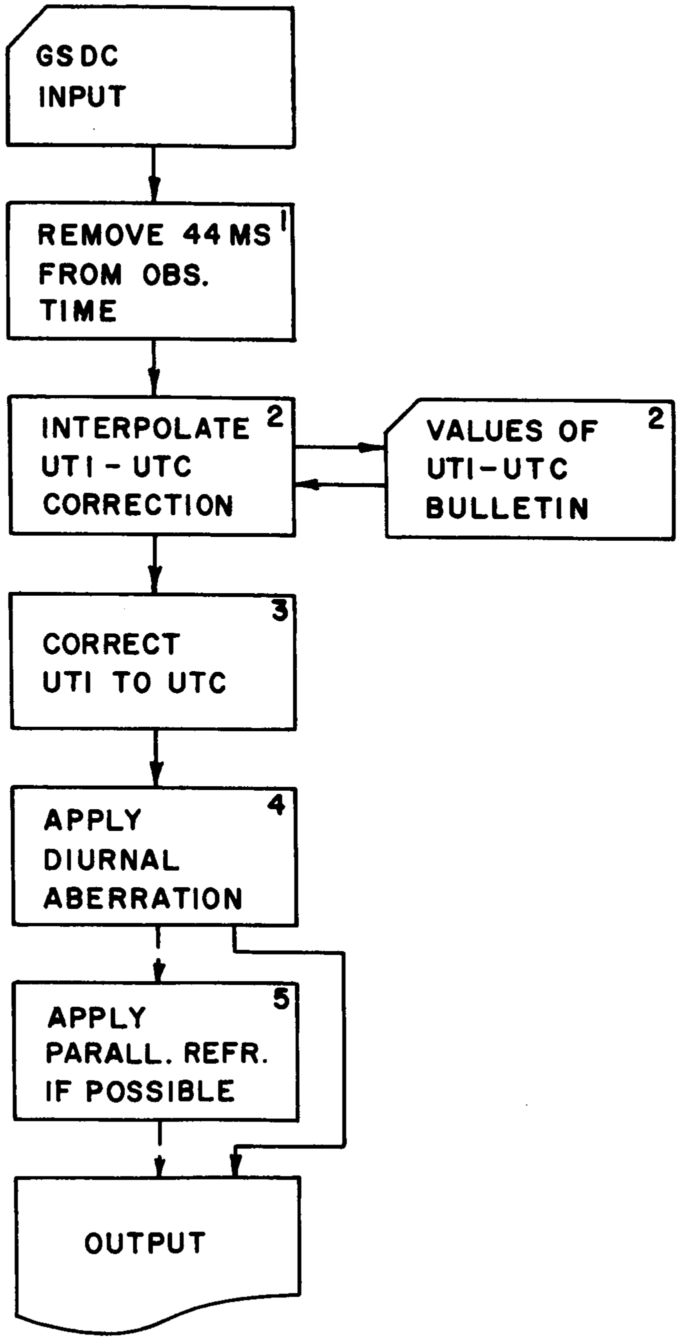
As mentioned in Section 5.1 the preprocessing procedures outlined in this section refer only to GEOS-A observations.

The ESSA GEOS-A results are submitted to the GSDC in the format given in Appendix I. The satellite's coordinates are given in the right ascension-declination system and represent the satellite's "apparent topocentric" position as explained in Section 4.45. The time of satellite observation refers to UT1 time obtained from preliminary U. S. Naval Observatory Bulletins to which 44ms has been added to refer the system to the old adapted longitude of the U. S. N. O.

As shown in Table 7, the satellite coordinates should be corrected for diurnal aberration and parallactic refraction. The epoch of observation should be corrected for the 44ms explained above (it should be removed), and UT1 should be corrected to UTC.

FIGURE 10

ESSA PREPROCESSING FLOW CHART FOR GEOS-A OBSERVATION



1. Remove 44 ms to correct from old longitude of the U. S. N. O. to current conventional longitude.

2. The next step is to "read in" the values of UT1-UTC published for ten-day intervals, and linearly interpolating the values to the date of observation to obtain the UT1-UTC correction. The UT1-UTC values to be "read in" should be taken from the "Preliminary Emission Times" Bulletin published by the U. S. Naval Observatory. The program must not allow interpolation if a step adjustment (in UTC) was made at one of the tabulated dates used for the interpolation. The Bulletin will indicate if a step adjustment was made. If it was made at one of the tabulated values used for interpolation, the author recommends using the other tabulated value without interpolation.

3. The next step is to correct the UT1 epoch to UTC. If the correction obtained in step 1 is less than 0. 5 seconds, it should be subtracted from the UT1 epoch. If greater than 0. 5 seconds, the correction should be subtracted from one and then added to the UT1 epoch.

4. Correct the satellite coordinates for diurnal aberration.

5. The next correction that should be made is to correct the satellite coordinates for parallactic refraction. Since the temperature and barometric pressure at the observing station are not available in the available GSDC format, the correction cannot be made.

5.24 The Aeronautical Chart and Information Center Preprocessing Procedures

The ACIC GEOS-A results are submitted in the GSDC format given in Appendix I. The satellite coordinates are given in right ascension-declination system and are referred to the true equator and equinox of the epoch of observation. The epoch of observation is the UTC time that the satellite was triggered to flash.

As indicated in Table 7, the satellite coordinates should be corrected for parallactic refraction. Unfortunately, as in the SAO case, the temperature and barometric pressure at the observing station are not available (in the available GSDC format) so the correction cannot be made.

As mentioned in Section 4.5, the ACIC uses the catalogued values of annual and secular variation from the Boss General Catalogue to update the star positions to the equator and equinox of the nearest Besselian Year. Technically, corrections should be applied to reduce the star positions from the Boss to the FK4 catalogue system, and to correct the star positions for third term variations. Unfortunately, this cannot be accomplished by an agency using the GSDC data since the stars used in plate reduction are not tabulated in the data output. Fortunately, the ACIC does plan to use the SAO catalogue in the future, so the catalogue system and third term variation problems will be eliminated.

5.3 FUTURE PREPROCESSING PROCEDURES

As mentioned in Section 5.24 and 5.32, complete homogenization of the data cannot be made because the temperature and barometric pressure at the observing station is not reported on the GSDC format, and, consequently, the parallactic refraction correction cannot be made. The author believes that an additional "observation" card could be used to circumvent the refraction correction difficulty and provide other data of interest to the data user. The proposed card should contain, at least, the following information (readily available from field records):

1. Identifying information to correlate the proposed card to the observation.

2. The temperature and barometric pressure at the observing station (available from the station logs) would allow users to apply parallactic refraction corrections to the satellite coordinates.

3. The geodetic coordinates of the station used in the plate reduction. The coordinates used must be given as explained in Section 3. 11 if future observation data is given in the azimuth-elevation system. The station coordinates can also be used in computing the range to the satellite for parallactic corrections.

4. If computed, the sidereal time of observation would also aid in data preprocessing; of course, if the satellite coordinates are given in the azimuth-altitude system, the sidereal time used must be known as explained in Section 3. 11. The sidereal time may also be used in determining the azimuth and altitude to the satellite for parallactic refraction corrections.

5. If the agency makes corrections to the U. T. C. epoch of observation, the precise amount of correction should be given. The satellite epoch correction to U. T. C. could then be made for further investigation by using the precise values the agency used.

6. The time signal source (i. e. , WWV or other transmission stations) must be known for accurate reductions of passive satellite observations. Presently, the SAO and ESSA procedures use portable clocks synchronized to transmitted WWV time signals for station synchronization; however, if observations

are used in the future from stations using only received time signal transmissions to determine the station epoch, the source of the time signals must be identified.

A very important question in the preprocessing of data from various agencies is how well will data from various agencies agree? Inter-comparison investigations have recently been made to attempt to ascertain the accuracy of the various systems used [Berbert, 1966]. A side-by-side test using seven different cameras (including the Baker-Nunn, MOTS, PC-1000 and BC-4) has recently been conducted at the SAO Baker-Nunn Camera site at Jupiter, Florida. The cameras were used to observe the GEOS-A satellite and the observations were reduced by the various participants. Preliminary results have been made available; however, they are not included in this report since the precise preprocessing methods were not given [Brown Associates, 1966, p. 3]. The author believes that the side-by-side results should be preprocessed by the methods outlined in Section 5.2 to determine first how well the results agree after preprocessing, and second, the practicability of programming the procedures.

APPENDIX I

FORMAT FOR THE NATIONAL GEODETIC SATELLITE PROGRAM OPTICAL OBSERVATIONS

The IBM card format given below is currently used for submitting optical satellite observations to the Geodetic Satellite Data Center by NASA, SAO, and ACIC. The format was taken from [Kahler, 1965].

FIELD	COLUMNS	DESCRIPTION
1	1 - 6	Satellite Identification As per COSPAR numbering system
	1 - 2	Year of Launch 64 = 1964 65 = 1965 66 = 1966 etc.
	3 - 5	Order of Launch
	6	Component Identifier 1 = a 2 = b 3 = c 4 = d etc.
2	7	Type of Coordinates 1 = Right Ascension and Declination 2 = Range 3 = Range Rate 4 = Frequency Shift 5 = Direction Cosines 6 = X, Y Angle 7 = Azimuth and Elevation Angle

FIELD	COLUMNS	DESCRIPTION
3	8	Observation Identifier 0 = Active (Observation on beacon) 1 = Passive (Chopping Shutter) 2 = Camera in conjunction with Laser 3 = Laser Angular data
4	9 - 11	Timing Standard Deviation
	9	Milliseconds
	10 - 11	.01 Milliseconds
5	12 - 13	Time Identifier 00 = UT-0 determined at observing station 01 = UT-1 determined at observing station 02 = UT-2 determined at observing station 03 = UT = C determined at observing station 04 = A. 1 determined at observing station 05-49 Other Systems 50 = UT-0 Satellite Time 51 = UT-1 Satellite Time 52 = UT-2 Satellite Time 53 = UT-C Satellite Time 54 = A. 1 Satellite Time 55-99 Other Systems
6	14 - 18	Station Number
	14	System Designator 0 = COSPAR 1 = AFCRL 2 = SAO 3 = STADAN 4 = TRANET DOPPLER 5 = AMS 6 = USCGS 7 = NAVAL OBSERVATORY 8 = INTERNATIONAL PARTICIPANTS
	15 - 18	Station Number

FIELD	COLUMNS	DESCRIPTION
7	19 - 34	GMT of Observation
	19 - 20	Year of Observation 64 = 1964 65 = 1965 66 = 1966 etc.
	21 - 22	Month of Observation
	23 - 24	Day of Observation
	25 - 26	Hour of Observation
	27 - 28	Minute of Observation
	29 - 30	Second of Observation
	31 - 34	.0001 Second of Observation
8	35 - 53	Observation Data R. A. (hours)/Azimuth degrees (arc), 0° North/ X angle (degrees arc). Sign of X angle appears in Column 35
	38 - 39	R. A. Minutes (of time)/Azimuth minutes (arc)/X angle .01 degrees (arc)
	40 - 41	R. A. seconds (time)/Azimuth seconds (arc)
	42 - 44	R. A. .001 seconds (time)/Azimuth .001 seconds (arc)
	45	Sign of declination/Y angle (+) (-)
	46 - 47	Declination, degrees (arc)/Elevation angle degrees (arc)/Y angle degrees (arc)
	48 - 49	Declination minutes (arc)/Elevation angle minutes (arc)/Y angle .01 degrees (arc)
	50 - 51	Declination, seconds (arc)/Elevation angle, seconds (arc)
	52 - 53	Declination .01 seconds (arc)/Elevation angle, .01 seconds (arc)

FIELD	COLUMNS	DESCRIPTION
9	54 - 59	Date of Plate Reduction
	54 - 55	Year of Reduction 64 = 1964 65 = 1965 66 = 1966 etc.
	56 - 57	Month of Reduction
	58 - 59	Day of Reduction
10	60 - 71	Coded Information
	60 - 61	Supplementary Documentation 03 = SAO Reduction Procedure Report 04 = MOTS Plate Reduction Procedure Report 05 = ACIC Plate Reduction Procedure Report 06 = USCGS Plate Reduction Procedure Report 07 = NASA Goddard R and R Preprocessing Report 09 = NASA Goddard Laser Preprocessing Report 10 = AFCRL LASER Reduction Procedure Report 11 = International Preprocessing Reports 12 = AMS Plate Reduction Report . (additional numbers will be assigned by NSSDC as required) . . n
	62 - 63	Equator Designation 01 = Mean Standard Equator 02 = Mean Equator at January 0.0 of Year of Observation 03 = Mean Equator at instant of observation 04 = Mean Equator at arbitrary time (arbitrary system to be defined in associated preprocessing report) 11 = True Standard Equator

FIELD	COLUMNS	DESCRIPTION
		12 = True Equator at January 0.0 of year of observation
		13 = True Equator at instant of observation
		14 = True Equator at arbitrary time (arbitrary system to be defined in preprocessing report)
	64 - 65	Equinox Designation
		01 = Mean Standard Equinox
		02 = Mean Equinox at January 0.0 of year of observation
		03 = Mean Equinox at instant of observation
		04 = Mean Equinox at arbitrary time (arbitrary system to be defined in associated preprocessing report)
		11 = True Standard Equinox
		12 = True Equinox at January 0.0 of year of observation
		13 = True Equinox at instant of Observation
		14 = True Equinox at arbitrary time (arbitrary system to be defined in associated preprocessing report)
	66 - 67	Instrumentation Type
		00 = PC-1000 MOD-1
		01 = PC-1000 MOD-2
		02 = BC-4 450 mm
		03 = BC-4 300 mm
		04 = BC-4 210 mm
		05 = Baker Nunn SAO
		06 = Baker Nunn - Military
		07 = MOTS
		08 = 1200 mm Ballistic Camera
		09 = 600 mm Ballistic Camera
		10 = MOTS 24"
		11 = International Types
	68 - 69	Catalog Identification
		01 = BOSS

FIELD	COLUMNS	DESCRIPTION
		01 = BOSS 02 = SAO Combined 03 = FK-4 04 = NASA Combined 05 = AGK-2 06 = AMS Combined 07 = Cape Zone, Vol. 1 09 = Others (to be defined in the associated preprocessing reports). Code number to be assigned by NSSDC
	70 - 71	Catalog Epoch 01 = 1855.0 02 = 1875.0 03 = 1900.0 04 = 1950.0 05 = 1965.0 06 = Others (to be defined in the preprocessing reports). Code numbers to be assigned by NSSDC
11	72 - 80	Description of Random Error
		Standard deviation in R. A. (seconds of arc) multiplied by the cosine of the declination/standard deviation in Az (seconds of arc)/ standard deviation in X angle (degrees in arc)
	73 - 74	Standard deviation R. A. (.01 seconds of arc) multiplied by the cosine of the declination/standard deviation in Az (.01 seconds of arc)/standard deviation in X angle (.01 degrees of arc)
	75	Standard deviation in declination (seconds of arc)/standard deviation in elevation angle (seconds of arc)/ standard deviation in Y angle (degrees of arc)
	76 - 77	Standard deviation in declination (.01 seconds of arc)/ standard

FIELD	COLUMNS	DESCRIPTION
		deviation in elevation angle (. 01 seconds of arc)/ standard deviation in Y angle (. 01 degrees of arc)
		Covariance; sign in column 78 (+) (-), decimal assumed between column 79 and 80

APPENDIX II

AVAILABLE COMPUTER PROGRAMS

As mentioned in the report, several computer programs and subprograms are currently available in The Ohio State University, Department of Geodetic Science which can be used in the preprocessing procedures. A brief description of the use of the programs is given.

The S 50 Program

Currently, the program is designed to generate data for geometric satellite triangulation studies. The program will compute and "punch" IBM cards with the following information:

1. Simultaneous directions from various ground stations to a satellite at a specified epoch.

2. Simultaneous ranges for the specified epoch.

3. A computed direction to the satellite by using a given station position and the satellite's orbital elements.

4. A computed range to the satellite by using the station position and the satellite's orbital elements.

Input data for the program consists of the following information:

1. The first card contains the datum reference number (in a two column integer field), a orbit reference number (in a two column integer field), and the length of semi-major and semi-minor axes of the reference ellipsoid

to be used (in two twelve column floating fields).

2. The second card contains two five column floating fields. The first field contains the standard error estimated for a direction observation (in seconds of arc). The second field contains the estimated standard error of a range observation (in meters). The standard error data is used to "generate" simulated data output using the estimated standard errors. For exact data output (required in preprocessing) the parameters should be set equal to zero.

3. The third set of cards contain the Smithsonian orbital elements for a specified orbit (see paragraph 1). The orbital elements are entered in the form,

$$X = X_0 + X_1 t + X_2 t + X_3 t + X_4 \cos \omega + X_5 \sin \omega .$$

The $X_0 \ldots, X_5$ are entered in a six field ten column format and represent the orbital elements' value at a time t from the initial epoch T_0. The coefficients used, in order, are,

ω	=	the argument of perigee of the orbit in degrees,
Ω	=	the right ascension of the ascending node in degrees,
i	=	the inclination of the orbit in degrees,
e	=	the eccentricity of the orbit (unitless),
M	=	the mean anomaly of the orbit in revolutions.

The final card (i. e., the sixth card) contains T_0 the epoch of perigee passage in Modified Julian Days in a one field ten column format.

4. The next set of cards consists of the geodetic station data. One card is required for each station used to "generate" the simultaneous data output. The maximum number of stations that can be used for one "run" is six, and a blank card must follow the last station card. The station data for each card is,

a) The station number in a form column integer field.

b) The datum number in a two column integer field (the number should be the same as in paragraph 1).

c) The station name (optional) in columns seven to twenty-four.

d) The sign of the latitude in a one column literal field.

e) The degrees of latitude in a three column integer field.

f) The minutes of latitude in a three column integer field.

g) The seconds of latitude in an eight column floating field.

h) The degrees of longitude (measured positive eastward for 0 to 360 degrees) in a three column integer field.

i) The minutes of longitude in a three column integer field.

j) The seconds of longitude in an eight column floating field.

k) The station elevation in meters in a twelve column floating field.

5. The final set of cards contains the epochs of observation used for simultaneous study. One epoch is entered per card in the following format, and the last card entered is followed by a blank card.

a) The hours of the epoch in a three column integer field.

b) The minutes of the epoch in a three column integer field.

c) The seconds of the epoch in an eight column floating field.

d) The day of the month in a three column integer field.

e) The month (in a three letter abbreviated form) in a three column literal field.

f) The last two digits of the year in a two column integer field.

g) The sighting number in a three column integer field.

The time of observation should be in the A. S. time system as described in Section 4.32 (Table 3).

The entire program outlined above could be used to generate the station to satellite range needed in Section 5 of the report; however, the author recommends that the program be "edited" for preprocessing optical satellite data. The edited version should accept a station's coordinates, epoch of observation, the satellite's orbital elements, and the time of observation. The program should then compute the geometric cartesian coordinates of the station (X, Y, Z) and the satellite (X', Y', Z') for the epoch in question. The range to the satellite (returned to the preprocessing program) may then be computed from

$$\text{Range} = \sqrt{(X'-X)^2 + (Y'-Y)^2 + (Z'-Z)^2} \quad .$$

The Star Updating Program

The author developed a star updating program to rigorously update star positions from the mean equator and equinox of 1950.0 to the apparent place referred to the equator and equinox of the epoch of observation. Statements in the program may be revised to provide the preprocessing procedures necessary for SAO data.

Input to the program consists of

1. A "time" card which contains the epoch of observation in the following order:

   a) The last two digits of the year of observation in a five column integer field.

   b) The month of observation (January is 1, February is 2, etc.) in a five column integer field.

   c) The day of observation in a five column integer field.

   d) The hour and minute of observation in two five-column integer fields.

   e) The second of observation in a five column floating field.

2. A "star information" card which contains the catalogued (i. e., 1950. 0 position) in the following order:

   a) The star's catalogue number in a five column integer field.

   b) The right ascension of the star (hour and minute) in two five-column integer fields and seconds in a ten column floating field.

   c) The declination of the star (degrees and mintues) in two five-column integer fields and seconds in a ten column floating field.

   d) The proper motions of the star (in right ascension and declination) are included after the "seconds" fields of the star position in paragraphs b and c above. The proper motion in right ascension (seconds of time) and declination (seconds of arc) are ten column floating fields.

   e) The Besselian Day Numbers C and D for the epoch of observation were obtained by interpolation (using second differences) from the Nautical Almanac and "fixed" in the program (by explicit statements).

The program essentially uses a method adapted from the artical by Scott and Hughes [Scott and Hughes, 1964]. A general description of the program is given below and subroutines which may be used in future preprocessing programs are given later.

The cartesian coordinates of the star's apparent position ($\underset{\sim}{X}$) are given by

$$\underset{\sim}{X} = \underset{\sim}{N}\ \underset{\sim}{\overline{P}}\ \underset{\sim}{A_y} + \underset{\sim}{N}\ \underset{\sim}{P}\ (\underset{\sim}{X_o} + \underset{\sim}{U}) \quad .$$

The vector $\underset{\sim}{X_o}$ represents the direction cosines of the star in the 1950.0 coordinate system; $\underset{\sim}{U}$ is the proper motion vector. The $\underset{\sim}{P}$ and $\underset{\sim}{N}$ matrices are the precession and nutation matrices from 1950.0 to the epoch of observation, respectively. The $\underset{\sim}{A_y}$ matrix is the aberration matrix formed from the Besselian Day numbers C and D (referred to the nearest Besselian Year) as shown below:

$$A_y = \begin{pmatrix} -D \\ C \\ C \tan \epsilon_y \end{pmatrix} \quad ,$$

where ϵ_y is the mean obliquity of the ecliptic at the nearest Besselian year. The P matrix "precesses" the aberration vector from the nearest Besselian year to the epoch of observation. The $\underset{\sim}{U}$ matrix is the proper motion matrix (explained in Section 2) evaluated at the epoch of observation.

Two subroutines used in the program are explained below:

1. The first subroutine (C. J. D.) will compute the Julian Date at 0^hU. T. for a given year (after 1957), month, and day. The subroutine designation (in

The Ohio State University SCATRAN language is

CALL SUBROUTINE (FD) = CJD. (KY, MO, JDY)

The "calling" parameters are KY, the last two digits of the year (since 1957); MO, the month; and, JDY the day of observation. The "returned" parameter is FD, the Julian Date minus 240,000.0 days.

2. The second subroutine (NUT.) will compute the nutation in obliquity ($\Delta\epsilon$) and longitude ($\Delta\psi$), for a given interval DN of Julian Ephemeris Days from 1900 January $0^{d}\!.5$ E. T. (J. E. D. 2415020.0). In SCATRAN the subroutine is

CALL SUBROUTINE ($\Delta\psi$, $\Delta\epsilon$) = NUT. (DN)

The subroutine uses the method outlined in the Explanatory Supplement to compute $\Delta\psi$ and $\Delta\epsilon$. Allen formulated and gives a statement listing of the program [Allen, 1966].

Statements and subroutines from the star updating program may be used to develop the preprocessing program outlined in Sections 2 and 5.

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For the Department of Geodetic Science

Project Supervisor Ivan I. Mueller Date 6/4/1967

For The Ohio State University Research Foundation

Executive Director Robert C. Stephenson Date 6/6/67